Hard and Soft Closing of Roads Towards Socially Optimal Routing

Jordan Ivanchev[1], Sorina Costache Litescu[2], Daniel Zehe[3], Mike Lees [4], Heiko Aydt[5], Alois Knoll[6]

Abstract—**Recent advances in Intelligent Transportation Systems, navigation tools and personal smart devices enable the development of effective mechanisms for improvement of traffic conditions. We present an information dissemination technique, which provides minimal but the right context to a population and steers the traffic system into a more efficient operational state. Selfish routing in large cities leads to a small group of roads being congested, while the rest of the road network remains underutilized [1], [2]. A routing steering mechanism is suggested, where we homogenize the traffic distribution by selectively disseminating information about the unavailability of certain roads, based on simulated outcomes of their closing. We demonstrate that the full removal of some road segments from the network can redistribute traffic in a socially beneficial way. We identify the most harmful roads and quantify their negative effect on the system. Furthermore, we introduce the concept of *soft closing*. Instead of informing the whole population to avoid a certain road, we inform only a portion of the drivers, further improving the network utilization. We use the city of Singapore as a case study for our traffic assignment model which we calibrate and validate using both survey and GPS tracking devices data.**

I. Introduction

The dynamics of the most intriguing and significant examples of large complex social systems are governed by a human factor. The presence of free will, in particular, leads to highly stochastic behavior, which can make modeling such systems challenging. The involvement of people in a system may further introduce a non-coordinated manner of operation. Those induced operational flaws, however, can be fixed by an efficient control approach.

In complex systems, inefficient states can be avoided by constructing adequate steering mechanisms [3]. Both the system's architecture, often represented by a network, and the time dependent dynamical interactions between the components make understanding and improving the performance of complex systems a challenging task. It has been shown that sparse inhomogeneous networks, which emerge in many real complex systems, are more difficult to control in comparison to dense and homogeneous systems [4].

As one example of complex systems, transportation systems are the subject of interest in variety of fields. There are many strategies of steering transportation systems that effectively increase traffic performance such as self-organizing traffic lights based on adaptation [5], [6], or information dissemination techniques as [7], [8], where commuters receive real time information about congestion in the network and adapt their routes accordingly.

The increasingly broader distribution of personal smart devices is a predisposition for the existence of Intelligent Transportation Systems (ITS). They progressively become more advanced [9] since the data availability provides a more complete view of the network, which leads to faster coordination [10]. Furthermore, since most drivers follow the advice provided by their navigation tool [11], the control of traffic can become more efficient and robust. The question stands, whether the system needs control at all. Drivers adapt to traffic conditions and tend to reach a Nash equilibrium state where a change of route for every participant would not be beneficial. In other words, the path choice of every driver is locally perceived as optimal and would not be voluntarily changed. This state of equilibrium is, however, not socially optimal when aiming at minimizing the overall population travel time. Our aim is to construct a coordinated information dissemination system that counteracts this trend.

A. Price of Anarchy

The phenomenon of non-coordinated social behavior guided by individual optimal strategies is studied in [12]. While every actor in a scenario has an individually optimal strategy, the collection of those strategies results in a socially sub-optimal performance of the system. This discrepancy between local and global strategy outcomes is called *price of anarchy* (POA) and indicates inefficiency due to lack of cooperation. In the case of transportation networks the standard uncoordinated behavior is called *selfish routing* and is studied in [13]. It, however, also exists in other complex networks such as the Internet [14].

Measuring and reducing the POA has been object to numerous studies such as [13] and [15]. Furthermore, [16] develops a useful general theory for bounding the POA in games of incomplete information, where players are uncertain about each others' pay-offs. Finally a middle ground between centrally enforced solutions and completely unregulated behavior is sought in order to achieve stability in [17].

B. Braess Paradox

Although unconventional, removing a road from the traffic infrastructure may lead to improved commuting conditions. The Braess paradox first mentioned in 1968 [18], states that adding extra capacity to a network where drivers act selfishly, can in some cases decrease performance. Evidence from 70 case studies from 11 countries that exhibit such conditions is presented in [19]. A generalization of the paradox [20] states that removing edges for large networks can produce an arbitrarily large improvement. It was further shown that the paradox can exist in all varieties of line-of-sight (LOS) networks as well [21]. Even the development of the human brain has a mechanism called synaptic pruning during which synapses (connections between neurons) are being removed in order to achieve more efficient learning [22]. It must be noted, however, that increasing the capacity of certain roads can lead to the avoidance of the paradox [23].

In [24] the paradox is analytically examined in the context of one-to-many and many-to-one OD matrices. The classical network used in [18] is used in [25] and the occurrence of the paradox is studied depending on the delay function parameters. In [26] a simple two path network is used and an addition of a link connecting the two paths is studied. Artificial small example networks are used to demonstrate the paradox in [27] and a routing algorithm based on collective information is developed that outperforms the shortest path algorithm reducing the harmful effects of the paradox. In [28] a technique for mitigation of the paradox is presented, which relies on providing recommendation based on traffic indicators.

Attempts for a detection methodology for the paradox can be found in [29]–[31]. In [32] it is shown, however, that the construction of Braess paradox free networks is NP hard and state that the paradox cannot be detected efficiently. In [33] a machine learning-inspired identification methodology for critical roads, which also include Braess roads, is described.

C. Contribution Summary

A trait of complex systems and their interaction is the emergence of the butterfly effect [34], where small changes in initial conditions can lead to performance alterations that are much bigger in magnitude [35]. We want to demonstrate that this holds true for traffic systems as well by removing a single road segment corresponding to one millionth of the size of a typical network. The effect of this modification on the systems performance in the sense of average travel time is then evaluated and compared in magnitude. Furthermore, we want to show that this change can, in fact, be beneficial for the system. In a way, we exploit the complexity of the system. First, we find the right road segment to remove using a brute-force search method (removing each link in turn and measuring the effect). Secondly, we try *soft closing* the road by informing a certain percentage of commuters that the road is closed.

Similarly to [8] and [36], in our study the control strategy is based on disseminating recommendations. Instead of providing information about traffic conditions, whose effects are highly unpredictable on a system level, we simply close a road for all users or just part of the population (*soft closing*). The choice of roads is not based on congestion levels but on the simulated outcome of those closures for the whole network. In this way the commuters are generically steered towards choosing more socially optimal routes and cases of local performance improvement that induce a negative effect on a global scale can be avoided. The readers may refer to the doctorate thesis of the first author [31] for an extended description of the work presented in this paper.

It is possible that, although well documented, in reality the Braess paradox may be stemming from secondary factors such as drivers taking less trips because of the reduced road network capacity. Also called disappearing traffic phenomenon [19], this translates in less overall usage of the road infrastructure. There seems to be no reasonable way to exclude the factor of willingness to travel when performing a real life experiment, which makes such empirical studies ambiguous. Furthermore, simulation based studies demonstrating the Braess paradox deal with artificial networks or just portions of real ones. In addition, a limited number of origin destination pairs are considered, thus making the results artificial. The chance that a road closure will be harmful to traffic conditions grows with increasing system size and generating authentic traffic that considers all participants and their diverse traffic demands, thus challenging the existence of the paradox in a realistic environment. We perform a complete city scale simulation, with systematic search of single road closure and provide a *soft closing* mechanism utilizing information dissemination tools to be able to dynamically control the system.

II. Data and Methods

In order to further study the examined phenomenon we perform a simulation based study, which allows us to control all factors in a systematic manner and state with certainty whether there are indeed harmful roads in a real world network scenario. By keeping the number of commuters and their origins and destinations constant within a single simulation run, we can isolate the phenomenon from all possible secondary influences and make sure that the measured changes in system performance are solely due to a change introduced in the network's topology.

Our case study examines the city of Singapore with population of 5.4 million people and around 1 million registered vehicles including taxis, delivery vans and public transportation vehicles [37]. It is an island city, which further simplifies our scenario since the examined system is relatively closed. We have used publicly available data to acquire a unidirectional graph of Singapore, that comprises of $240,000$ links and $160,000$ nodes representing the road system of the city. The number of lanes, speed limit and length of every link is available allowing us to extract information about its capacity.

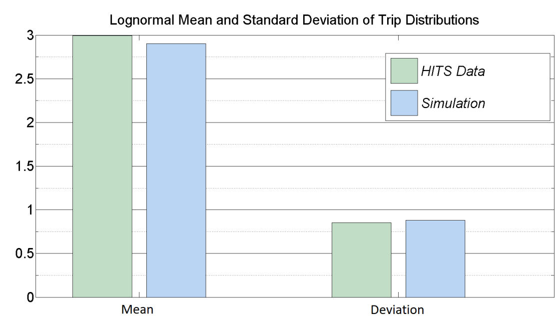
For the purposes of our model's calibration and validation we make use of two separate data sets. The first one is the Household Interview Travel Survey (HITS) that covers 0.67% of the population of Singapore conducted in 2012, which studies the traffic habits of the population. Information about the origin destination pairs, their temporal nature, and commuting time distribution during rush hour periods is extracted from it. The second data set consists of GPS trajectories of a $20,000$ vehicle fleet for the duration of one month, providing information about recorded velocities on the road network during different times of the day.

Realistic traffic is modeled by utilizing the macroscopic traffic assignment of the SEMSim simulator described in more detail in [31]. A sufficiently large vehicle population is generated based on the HITS dataset. Every driver is first assigned an origin and destination sampled from the OD data using Bayesian estimation with assumed uniform prior distribution. After that route preferences are assigned, based on speed, distance or comfort, with chosen probabilities. After all the routes have been computed the flows F_i along every road segment can be extracted.

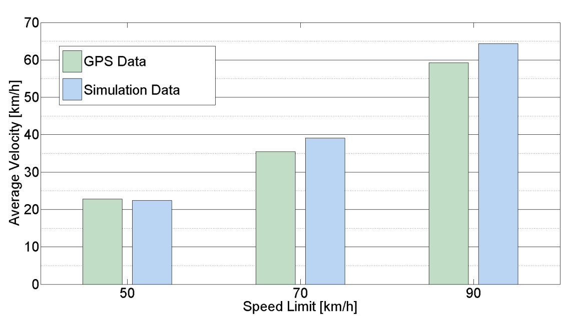
A variation of the Bureau of Public Roads (BPR) function [38] is used to compute travel times along every road segment i depending on the flow of vehicles F_i:

$$t_i = \min\left(\frac{l_i}{v_f^s}\left(1+\alpha^s\left(\frac{F_i}{2000 w_i t}\right)^{\beta^s}\right), \frac{l_i}{v_{min}}\right) + I(i)d^s$$

where l_i is the length of the road segment,v_f is the free flow velocity, w_i is the number of lanes, t is the simulation time from which the flow is calculated in hours, v_{min} is the minimum flow velocity at the link at extreme congestion levels, $I(i)d^s$ accounts for time loss due to intersections or on ramps depending on the number of incoming roads $I(i)$, and the subscript s indicates the speed category of the link. The additional delay d^s is different for the distinct types of roads and is calibrated later on.



(a) Lognormal mean and deviation of trip duration



(b) Average speed for different road types

Fig. 1: Comparison between real data and calibrated simulation.

Free flow velocities v_f are extracted from GPS tracking data. Parameters α^s and β^s are calibrated for different types of roads depending on their speed limits. The calibration criterion was to minimize the difference between 1) simulated and extracted distribution of travel times from the HITS dataset and 2) simulated and extracted average speed along road category types utlizing the GPS tracking data. The comparison between the real life data and the calibrated simulation can be seen on Fig. 1. The validation of the traffic assignment was then performed based on comparing the traffic states among selected critical road segments from the GPS tracking data. The results of the validation procedure demonstrate that for the desired level of detail (average traverse time along roads) our approach produces results which are satisfactory close to reality.

III. Study 1: Road Removal

The initial experiment aims at identifying links (road segments) whose closure would result in better traffic performance. All $240,000$ road segments are removed consecutively from the routing graph. After every removal a new traffic assignment is performed and the new population travel times are computed. The results are compared to the initially simulated scenario, while the origins and destinations of all drivers are kept the same for all simulation runs. The most computationally expensive operation in the implementation of this systematic search is running the shortest path algorithm on a large graph. Roughly 21 million routings have been performed for this study. We compute the shortest path by utilizing the contraction hierarchy algorithm [39], which takes 50 microseconds on average on the Singapore graph. This makes up for 1000 seconds CPU time to run the whole experiment. The presented approach can be easily parallelized, which allowed us to run it on 32 threads reducing the runtime of the experiment to 30 seconds.

This procedure allows us to systematically evaluate the effect of a road segment closure. Our results confirm that in the examined case study closing a road segment can have a positive effect on the total travel time within the system. In 21 cases the closure of a road segment leads to a decrease of 1 minute or more in the average travel time. This accounts to to 3.73% system-wide travel time decrease. The most harmful link gives a 74.25 seconds decrease of overall trip duration translating to 6400 saved hours for the driver population on a daily basis, solely from the morning rush hour period. Although part of the backbone of the network in a topological sense, the removal of certain major road segments, would decrease overall travel time. In other cases, however, removing such important roads significantly increases commuting time for the population. Those links are identified as crucial for the traffic system.

The effects of beneficial road closure are illustrated in Fig. 2. It can be observed that the length of the roads that receive additional traffic (colored in red) is greater than the length of the roads with decreased flows (green). However, we know for a fact that the overall travel time in the system is reduced. This means that the time saved from traffic going away from the green roads is more than the time gained from the additional vehicles on the red roads. This is possible due to the non-linearity of the speed-flow relationship and it basically means that agents were taken from highly congested roads and distributed among less traffic-intense areas thus alleviating overall traffic congestion.

Typically, when a road closure occurs it forces detours along longer paths, and possibly moves traffic from a large congested roads to smaller congested roads, which effectively makes the congestion even worse. In fact, this is what happens in 99.73% of the experiments that we have run. This results is rather intuitive as decreasing the connectivity in a large complex system would typically have negative impact on the system. For the closure 64 road segments, however, this intuition is not correct.

There are two main reasons which explain why this phenomenon occurs. First, the price of anarchy is somehow reduced for the closure of certain roads. It seems that in highly rare cases the closure of a specific road, forces selfish harmful drivers into more socially optimal routes while not causing significant amount of damage. Second, the road network itself allows for high level of price of anarchy. As the road network evolves gradually, it is not optimally designed for the current demand in the system, thus it allows for the existence of the Braess paradox. If a road network is designed from scratch for a given demand, guarantees that there are no Braess roads can be provided.

Fig. 2: Traffic pattern changes due to road closure.

While closing a road seems to successfully redistribute traffic in a few cases, such an extreme measure might not bring the biggest possible benefit to the population. Sometimes a driver, might have only one viable route option and removing it would create a significant inconvenience for such drivers. Therefore, in the next section we look at a milder strategy, which makes the road inaccessible to only a portion of the drivers thus allowing the routes of the population to be steered more efficiently.

IV. Study 2: Soft Closing

We suggest the concept of *soft closing* of links. Rather than removing a link from the road network completely, we remove it only for a fraction of the agents that initially pass through it. In this way traffic demand in the city is re-distributed more homogeneously. An extension of our initial experiment is performed where instead of informing all agents that a certain road is unavailable we do so only for half of the drivers passing through it. The experiment is done in order to examine whether partial closing of certain links can further decrease overall travel time. The figure of 50% closure was chosen since it is the middle ground between completely closed and opened. We refer to drivers that cannot pass through the link anymore as *informed* and the to the ones that can stay as *uninformed*. Since the two groups are chosen at random, we perform the experiment for every link 10 times in order to evaluate the effects of informing different subgroups.

We have found 64 links whose partial closure leads to more than 1 minute decrease of average trip duration. The link that shows best performance if partially closed gives 100 seconds decrease of the population average travel time. This amounts to 6.25% increase in system-wise performance. It must be noted that the links, which lead to biggest improvement if completely closed and those who are only half closed do not coincide. Only 2 out of the 64 links with best performance from *Study* 2 are in the list of harmful links from *Study* 1, which indicates an underlying categorization of roads according to their optimal flows.

In order to evaluate the induced system effects arising from the variation of percentage of informed agents, we examine in detail the top two links representing different roads from each of the sets of harmful road segments from our two previous experiments. We perform a sweep of the percentage of *informed* agents and for every step evaluate the average trip duration for the population. The resulting graph (Fig. 3) provides an overview of the effects of changing the accessibility of a link to the whole system. The curves of the links that come from *Study* 1 (Road 1 and 2) reach their minimum in proximity to 100% and resemble a linear function. The links coming from *Study* 2(Road 3 and 4) have convex curves with optimal percentage of redirected agents between 40% and 50%. In the latter cases by further reducing the traffic on the selected links the system's performance starts to deteriorate due to other congestion spots created as a result of the traffic re-distribution. Fig. 3 also provides a reference level by showing the system's optimum performance computed using the BISOS algorithm [40].

Does the choice of agents that are not allowed on the link affects the results? For every road closure experiment a different set of agents is informed and redirected. Therefore, variations in the results are to be expected. The computed coefficient of variation σ / μ on those experiments is 4×10^{-4}. It can be noted the deviation is surprisingly small, which means that the choice of agents, which need to find other routes is not a decisive factor. This simplifies significantly the analysis of our *soft closing* strategy. This unexpected discovery may be explained with the fact that by considering a real world scenario we also get a great variety of origin destination pairs. The apparent homogeneity of agents on this level of

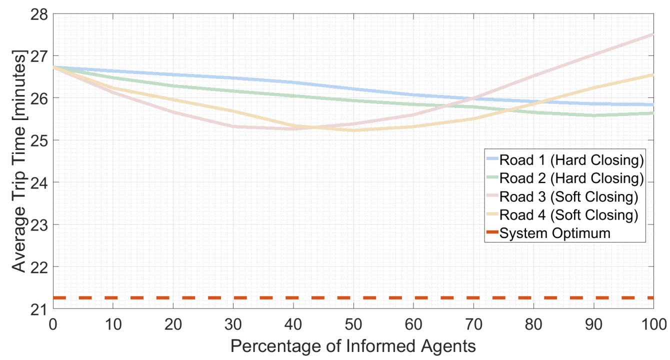


Fig. 3: Variation of the average population travel time for different percentages of "soft" closing of examined links.

abstraction thus allows us to consider them as groups rather than individuals.

V. Study 3: Local Effects and Price of Anarchy

As a final step, the implications for affected agents are studied for the four previously examined links. Fig. 4 displays how the *informed* and *uninformed* agents perform for different degrees of closure of links. Examining travel times at the point of social optimum, the groups of *informed* agents save between 23% and 41% travel time, while the *uninformed* agents benefit the reduced congestion on the initial path and get between 23% and 50% improvement. This shows that none of the primary affected groups of agents experiences negative effects. On the contrary, the improvements in their average travel times are between 4 and 8 times higher than the overall population performance increase.

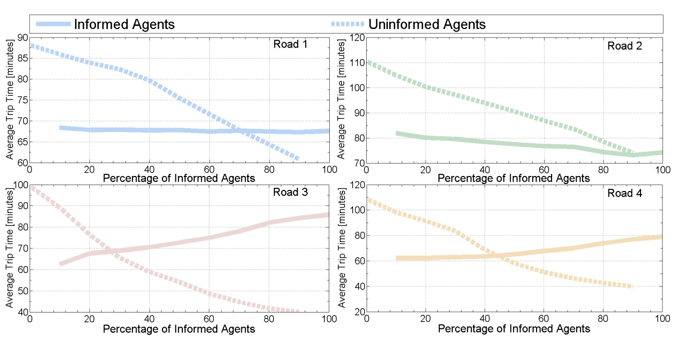


Fig. 4: Comparison of the average travel time of informed and uninformed agents for different percentages of *soft closing*.

Given the invariance of our results to the *informed* agents selection, it is expected that the curve of *uninformed* agents has a negative slope, since congestion levels along their paths decrease. It can be noticed that the *informed* and *uninformed* agents' curves cross on Fig. 4. At the point of crossing it can be assumed that an agent who has perfect information about the traffic situation would not make the choice to change from *informed* to *uninformed* or vice versa. This point can be perceived as a Nash equilibrium for this link for a single commuter.

Furthermore, the point of equilibrium for the collective group of *informed* and *uninformed* agents, which can be considered as a local social optimum, is also identifiable by locating the minimum of the affected agents group average travel time function. The three points of interest (single agent equilibrium, affected agents group equilibrium and social optimum) do not coincide in the 3 out of the 4 studied cases as seen on Table I. The desired percentage of closure that should be chosen in general is the percentage for social optimum to occur since it saves the biggest amount of total time. If, however, agents choose their routes selfishly or even in local groups a different equilibrium point will be reached, thus leading to a sub optimal traffic distribution resulting in society paying the POA due to lack of coordination.

It must be noted that this discrepancy does not result from lack of information. The alternatively calculated personal Nash equilibrium and group equilibrium are based on full knowledge of the system. We can thus conclude that simply choosing the fastest route even in the presence of perfect information does not lead to optimal traffic distribution. It is, therefore, vital that the system is always considered as a whole because the collection of local optimal solutions may not produce the expected result due to the high complexity.

Road	Social Optimum [%]	Nash Equilibrium [%]	Affected Group Optimum [%]
Road 1	100	70	90
Road 2	90	90	90
Road 3	40	27	50
Road 4	50	44	60

TABLE I: Points of Equilibrium

VI. Conclusion

In the context of fast changing topologies of complex networks, this work examines the phenomenon of removing parts of a graph to improve the overall system's performance. This allows for effective steering mechanisms that introduce small changes in the system and achieve results of much greater magnitude, thus exploiting the systems's complexity. The concept of *soft closing* uses information as a steering tool in order to turn the previously static road infrastructure into a dynamically changing intelligent transportation system. We consider this a first step towards eliminating price of anarchy phenomena induced by lack of coordination.

In the presented research, we have confirmed that by disseminating information about a road that should be avoided to all traffic participants on a large city scale the system's performance can be improved. We show an example for a given OD matrix which demonstrates how the closure of a single road in a large city can lead to 4% reduction of travel time. By disseminating the information about the closure only partially to the population we perform *soft closing* of roads in order to further improve traffic conditions (up to 6%). This strategy provides the ability of the road network to behave dynamically, at zero infrastructure construction cost, via information dissemination.

Simulation based methods such as *soft closing* can be used to ensure efficient utilization of resources and fast instantaneous adaptability to demand changes. It is important to study further such information dissemination techniques since they provide flexibility and dynamic properties to the physically static road infrastructure, a trait that is highly desired in the future of transportation.

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