Decentralized Voltage Support in a Competitive Energy Market

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***Abstract*—In order to facilitate the increasing amount of distributed generations and flexible loads, it is envisioned that the future power grid might operate in a more decentralized manner. To this end, this work proposes a decentralized market framework for multiple interconnected microgrids which can exchange energy and provide voltage support. In contrast to the state-of-the-art analysis, we consider both the energy and the voltage support as the commodities which can be exchanged between the microgrids. We use alternating direction method of multipliers (ADMM) as a market clearing mechanism which obtains market clearing prices for the proposed commodities, while considering grid constraints. Moreover the proposed mechanism promotes the operation autonomy of the involved entities by safeguarding their information privacy. We test the proposed framework on a modified 144-bus network which is partitioned into 5 microgrids.**

***Index Terms*—ADMM, distributed generators (DG), voltage support market**

I. Introduction

Driven by decarbonization, electrification in transport sector, grid deregulation and many other driving forces, power system is evolving from a centralized system which supplies from a few generation facilities to the passive loads towards an intelligent and active system consisting of multiple cyber-physical layers. Deregulation of the energy market can potentially create a better social welfare [1]. Deployment of the high-speed communication network enables the interactive play of cyber-physical system, creating opportunities to integrate more market participants in a liberalized market. Technologies like Internet of Things (IoT) and Blockchains provide solutions for new transaction possibilities [2]. To reduce the investment cost for installing more necessary generation capacity, demand side management is becoming more relevant [3]. The integration of renewables and distributed energy resources (DER) comes with the evolving control architectures, requiring a more cost-effective management and new market framework. Decentralized grid operations show the supremacy against centralized grid operation in terms of cybersecurity, robustness in management, investment reduction [4], [5]. Therefore, an increasing degree in operation decentralization necessitates a new market mechanism to maintain the market equilibrium without violating the network constraints.

As identified in many previous works [1], [6], distributed control and decentralized market design are the two cornerstones in enabling the active microgrid. Work [7] provides the solution for the distributed energy storage system management in a multiple microgrid scenario. A distributed market framework generating real-time price signals based Karush-Kuhn-Tucker (KKT) conditions is proposed in [8]. Work [9] focuses on energy import and export schemes for a similar grid organization with the grid consisting of a number of microgrid communities while the energy is traded within/between the community under a supervisory node. These market frameworks, however, come with drawbacks that the network constraints are neglected. Work [9] is restricted in its geographical preferences without taking into account of voltage stability. Indeed, due to the nature of AC microgrids, voltage support becomes one of the major challenges in the regulation. Current market framework [7]–[9] in the literature does not consider the voltage support as the services can be traded within the smart grid community. In fact, for the multiple microgrids scenario, the local voltage can have a positive impact on the connected neighboring microgid, if the voltage controlled properly. We exploit such a voltage support service in this work.

The aim of the work is to provide a decentralized market framework for voltage support. For each microgrid, we consider the procurement of the active/reactive power as well as the coupling voltage in the inter-microgrid trade. The proposed trade problem is equivalent to a optimal power flow (OPF) problem which can be solved by ADMM efficiently. The methodology based on ADMM is shown as a process of working towards the market equilibrium with the dual variables of ADMM obtained as the market equilibrium price. The prices generated by ADMM are further analyzed in the test case study.

Notations: $\mathbb{R}$ and $\mathbb{C}$ denote the set of real and complex numbers. Scalars are small letters, i.e., x. Vectors and matrices are in bold letters, i.e., $\mathbf{x}, \mathbf{X}$. Entries of a matrix $\mathbf{X}$ are specified by x_{ij}. Entries of vector $\mathbf{x}$ are specified by x_i whereas the regional version of $\mathbf{x}$ is given as $\mathbf{x}_i$. Conjugates of a complex scalar, vector or matrix are denoted by $\underline{x}, \underline{\mathbf{x}}, \underline{\mathbf{X}}$. The obtained optimal solutions are denoted as $x^*, \mathbf{x}^*$. For complex scalars, vectors or matrices, $\Re(), \Im()$ are used to extract the real and imaginary part. The transpose of a vector or matrix is denoted by $()^\intercal$ and $\mathrm{diag}(\mathbf{x})$ constructs a diagonal matrix with entries of $\mathbf{x}$.

II. Problem setup

A. Linearized DistFlow Model

We consider a group of interconnected microgrids at the medium or low voltage level in Fig. 1. The smart grids are included in set $\mathcal{S} := \{1, 2, ..., s\}$. The individual microgrid is assumed to have a distribution grid nature with a reference bus indexed by 0, which is modeled as a slack bus. The rest of the buses are modeled as PQ buses. All grid buses are included in set $\mathcal{N} := \{0, 1, 2, ..., n\}$. In addition, we define set $\mathcal{L} := \{1, 2, ..., h\}$ containing all the lines connecting the buses. A line is denoted as $(i, j) \in \mathcal{L}$. The complex nodal injections and voltages are represented for all buses as $\mathbf{s} := \mathbf{p} + \mathrm{j}\mathbf{q}$ and $\mathbf{u} := \mathbf{v} \cdot \mathbf{e}^{\mathrm{j}\boldsymbol{\theta}}$ all with size $\mathbb{C}^{n+1}$. The individual buse $i \in \mathcal{N}$ defines $s_i := p_i + \mathrm{j}q_i$ and $u_i = v_i e^{\mathrm{j}\theta_i}$. Given a branch (i, j) connecting bus i and j, the network branch flow can be described using the LinDist flow model [10].

$$p_j = p_{ij} - r_{ij}l_{ij} + \sum_{(i,j)\in\mathcal{L}} p_{jk} \tag{1}$$

$$q_j = q_{ij} - x_{ij}l_{ij} + \sum_{(i,j)\in\mathcal{L}} q_{jk} \tag{2}$$

$$v_j^2 = v_i^2 - 2(r_{ij}p_{ij} + x_{ij}q_{ij}) + (r_{ij}^2 + x_{ij}^2)l_{ij} \tag{3}$$

$$l_{ij} = \frac{p_{ij}^2 + q_{ij}^2}{v_i} \tag{4}$$

with $p_{ij}, q_{ij}, l_{ij} \in \mathbb{R}$ denoting the branch active/reactive flow and the line current magnitude. r_{ij}, x_{ij} are the line resistance and reactance respectively. Now neglecting the higher-order loss terms $r_{ij}l_{ij}, x_{ij}l_{ij}$ and $(r_{ij}^2 + x_{ij}^2)l_{ij}$ in (1) to (3) and applying a flat voltage profile approximation that $v_j^2 - v_i^2 = (v_j + v_i)(v_j - v_i) = 2(v_j - v_i)$, we obtain

$$p_j = p_{ij} + \sum_{(i,j)\in\mathcal{L}} p_{jk} \tag{5}$$

$$q_j = q_{ij} + \sum_{(i,j)\in\mathcal{L}} q_{jk} \tag{6}$$

$$v_j = v_i - (r_{ij}p_{ij} + x_{ij}q_{ij}) \tag{7}$$

which is a linearized DistFlow model. The linearized DistFlow model has shown to perform reasonably well, i.e., the approximate voltage magnitude error is under 0.25% for the voltage deviation of 5% from the nominal [11]. We use the resistance and reactance matrix $\mathbf{R}, \mathbf{X} \in \mathbb{C}^{(n+1)\times(n+1)}$ to model the grid, hence we can rewrite the above equation (5) to (7) in a compact form as:

$$\mathbf{v} = \mathbf{v_0} + \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} \tag{8}$$

where $\mathbf{v_0} \in \mathbb{R}^{n+1}$ is a vector with all the entries equal to v_0 (slack bus).

B. Multiple Microgrids Market for Voltage Support

We now illustrate the proposed grid organization in Fig. 1. Consider a decentralized grid operation consisting of multiple microgrids, each microgrid is operated by a local micro grid operator (MGO) who plays a supervisory role in monitoring inter-microgrid transactions as well as its own microgrid's operation. Each microgrid has local generations and import/export capabilities based on physical connections to other microgrids. In this regards, the local MGO is responsible for clearing inter-microgrid trading, which is basically minimizing the local operation cost while satisfying the local energy demand and maintaining the allowable voltage levels. It is assumed that all market participants are economically rationale, price takers and submit their true marginal generation curve the local MGO.

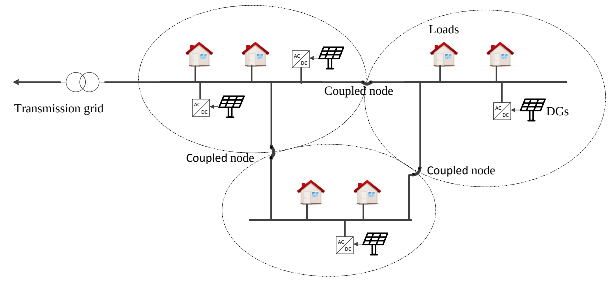

Fig. 1. Multiple microgrids scenario

For the sake of brevity, if a local microgrid connected to transmission grid over power supply point (PSP), the marginal cost of power delivered to the local microgrid from PSP is simply modeled through the respective locational marginal price (settled by transmission system operator (TSO)). Hence, the PSP can be considered as a distributed generation supplying energy to the local microgrid. Let $\mathbf{p}_i^{\mathrm{g}}, \mathbf{q}_i^{\mathrm{g}} \in \mathbb{R}^{g_i}$ denote the distributed generations in microgrid $i \in \mathcal{S}$, giving the total cost of energy generations:

$$c_i(\mathbf{p}_i^{\mathrm{g}}, \mathbf{q}_i^{\mathrm{g}}) = \mathbf{c}_i^{\intercal}\mathbf{p}_i^{\mathrm{g}} + \mathbf{d}_i^{\intercal}\mathbf{q}_i^{\mathrm{g}} \tag{9}$$

where $\mathbf{c}_i/\mathbf{d}_i \in \mathbb{R}^{g_i}$ is the generation price vectors for active/reactive power procurement. Aiming for the voltage support through a market, an additional voltage regulation term $w_i(\mathbf{v})$ is considered as the penalty cost of voltage deviation to the nominal voltage $\mathbf{v}_i^{\mathrm{norm}} \in \mathbb{R}^{n_i}$

$$w_i(\mathbf{v}) = (\mathbf{v}_i - \mathbf{v}_i^{\mathrm{norm}})^{\intercal}\mathrm{diag}(\mathbf{w})(\mathbf{v}_i - \mathbf{v}_i^{\mathrm{norm}}) \tag{10}$$

where $\mathbf{w} \in \mathbb{R}^{n_i+1}$ sets the weighting factor for the voltage deviation cost to the generation costs [4].

Since the microgrids are interconnected, each grid has coupled buses shared with other grids that they can exchange power (e.g., the coupled buses connecting two neighboring microgrids in Fig. 13). All the coupled buses for microgrid i are included in set $\mathcal{C}_i := \{1, 2, ...m_i\} \in \mathbb{R}^{m_i}$. Furthermore, the power exchange at these coupled nodes are denoted as $\mathbf{p}_i^{\mathrm{exc}}, \mathbf{q}_i^{\mathrm{exc}} \in \mathbb{R}^{m_i}$ with its voltage denoted as $\mathbf{v}_i^{\mathrm{exc}} \in \mathbb{R}^{m_i}$. Hence the nodal injections $\mathbf{p}_i, \mathbf{q}_i \in \mathbb{R}^{n_i+1}$ in microgrid i are the collections of its generations $\mathbf{p}_i^{\mathrm{g}}, \mathbf{q}_i^{\mathrm{g}}$, power exchange (import minus export) and loads $\mathbf{p}_i^{\mathrm{load}}, \mathbf{q}_i^{\mathrm{load}} \in \mathbb{R}^{n_i+1}$:

$$\mathbf{p}_i = \mathbf{A}_i\mathbf{p}_i^{\mathrm{g}} + \mathbf{B}_i\mathbf{p}_i^{\mathrm{exc}} - \mathbf{p}_i^{\mathrm{load}} \tag{11}$$

$$\mathbf{q}_i = \mathbf{A}_i\mathbf{q}_i^{\mathrm{g}} + \mathbf{B}_i\mathbf{q}_i^{\mathrm{exc}} - \mathbf{q}_i^{\mathrm{load}} \tag{12}$$

with $\mathbf{A}_i \in \mathbb{R}^{n_i\times m_i}$ is the mapping matrix from generation buses to PQ buses whereas $\mathbf{B}_i \in \mathbb{R}^{n_i\times g_i}$ is the mapping matrix from the coupled buses to PQ buses.

For a microgrid i, all its physically connected neighboring microgrid are included in set $\mathcal{N}_i$. In particular, the power exchange with $j \in \mathcal{N}_i$ and the local microgrid i is defined as $\mathbf{M}_{ij}\mathbf{p}_j^{\mathrm{exc}}$. The incidence matrix $\mathbf{M}_{ij} \in \mathbb{R}^{m_i\times m_j}$ projects the coupled bus of microgrid j to i. Furthermore, the active/reactive power exchange between microgrids holds the energy conservation rule:

$$\mathbf{p}_i^{\text{exc}} = -\sum_{j\in\mathcal{N}_i} \mathbf{M}_{ij}\mathbf{p}_j^{\text{exc}} \tag{13}$$

$$\mathbf{q}_i^{\text{exc}} = -\sum_{j\in\mathcal{N}_i} \mathbf{M}_{ij}\mathbf{q}_j^{\text{exc}} \tag{14}$$

Moreover, the voltage at the coupled buses are related as:

$$\mathbf{v}_i^{\text{exc}} = \mathbf{M}_{ij}\mathbf{v}_j^{\text{exc}} \qquad \forall j \in \mathcal{N}_i \tag{15}$$

C. Problem statement

To this end, for each local MGO, following operation cost minimization problem applies

$$\min \quad f_i(\mathbf{p}_i^{\text{g}}, \mathbf{q}_i^{\text{g}}) := c_i(\mathbf{p}_i^{\text{g}}, \mathbf{q}_i^{\text{g}}) + w_i(\mathbf{v}_i) \tag{16a}$$

s.t.

$$\mathbf{v}_i = \mathbf{v_{0,i}} + \mathbf{R}_i\mathbf{p}_i + \mathbf{X}_i\mathbf{q}_i \tag{16b}$$

$$\underline{\mathbf{p}}_i \leq \mathbf{p}_i \leq \overline{\mathbf{p}}_i \tag{16c}$$

$$\underline{\mathbf{q}}_i \leq \mathbf{q}_i \leq \overline{\mathbf{q}}_i \tag{16d}$$

$$\underline{\mathbf{v}}_i \leq \mathbf{v}_i \leq \overline{\mathbf{v}}_i \tag{16e}$$

$$(13) - (15)$$

where the operation cost in (16a) is written as the minimum of a weighted linear combination of generation cost and the voltage deviation [4]. Constraints (16c) to (16e) are the box constraints for power dispatches and voltage magnitude. Constraint (16b) is the regional linearized DistFlow constraint whereas constraints (13) and (14) ensure the power supply from neighboring microgrids meets the local import demand. Constraint (15) ensures the coupling voltages being the same.

Since each microgrid, although connected with other microgrids, wishes to operate autonomously, a market mechanism without a central coordinator is needed to coordinate the solution of (16). This market mechanism then needs to clear the price for energy exchange $\mathbf{p}_i^{\text{exp}}, \mathbf{q}_i^{\text{exp}}$ and coupled bus voltages $\mathbf{v}_i^{\text{exc}}$ between the connected microgrids. Note that even though the voltage on the coupled bus is not conventionally a trade-able commodity, we show that by adjusting it through a price mechanism improves the overall grid's voltage profile. This is because due to the AC nature of the microgrid the voltage at the local bus is coupled to its neighboring buses' actions. Hence, by allowing these coupling voltages to have an economic value, this provides an incentive to support voltage improvement actions. In the next section, we present ADMM as a market mechanism for exchanging energy between microgrid communities while archiving the market equilibrium.

III. Decentralized Market Clearing with ADMM

In general, ADMM provides a generic algorithm for solving optimization problem in a distributed fashion. We introduce the key ingredients of ADMM as follows. Consider problem (16), if each local MGO optimizes the local problem and obtain the results for $\mathbf{p}_i^{\text{exc}}, \mathbf{q}_i^{\text{exc}}, \mathbf{v}_i^{\text{exc}}$, the connected microgrid j may not satisfy the desired import demand with its supply capability $\mathbf{M}_{ij}\mathbf{p}_j^{\text{exc}}$. Hence a set of global variables are defined here that can be interpreted as the stepwise trade-off results for $\mathbf{p}_i^{\text{exc}}, \mathbf{q}_i^{\text{exc}}, \mathbf{v}_i^{\text{exc}}$ after negotiation with other MGOs. The global variables are represented as $\mathbf{p}_i^{\text{exc,+}}, \mathbf{q}_i^{\text{exc,+}}, \mathbf{v}_i^{\text{exc,+}}$. Furthermore, we define the local augmented Lagrangian for microgrid i as:

$$\begin{aligned} L_i^{\text{admm}} =& c_i(\mathbf{p}_i^{\text{g}}, \mathbf{q}_i^{\text{g}}) + w_i(\mathbf{v}_i) + \boldsymbol{\lambda}_i^{\intercal}(\mathbf{p}_i^{\text{exc}} - \mathbf{p}_i^{\text{exc,+}}) \\ &+ \boldsymbol{\kappa}_i^{\intercal}(\mathbf{q}_i^{\text{exc}} - \mathbf{q}_i^{\text{exc,+}}) + \boldsymbol{\sigma}_i^{\intercal}(\mathbf{v}_i^{\text{exc}} - \mathbf{v}_i^{\text{exc,+}}) \\ &+ \frac{1}{2}\rho_i(\mathbf{q}_i^{\text{exc}} - \mathbf{q}_i^{\text{exc,+}})^{\intercal}(\mathbf{q}_i^{\text{exc}} - \mathbf{q}_i^{\text{exc,+}}) \\ &+ \frac{1}{2}\rho_i(\mathbf{p}_i^{\text{exc}} - \mathbf{p}_i^{\text{exc,+}})^{\intercal}(\mathbf{p}_i^{\text{exc}} - \mathbf{p}_i^{\text{exc,+}}) \\ &+ \frac{1}{2}\rho_i(\mathbf{v}_i^{\text{exc}} - \mathbf{v}_i^{\text{exc,+}})^{\intercal}(\mathbf{v}_i^{\text{exc}} - \mathbf{v}_i^{\text{exc,+}}) \end{aligned} \tag{17}$$

Variables $\boldsymbol{\lambda}_i, \boldsymbol{\kappa}_i, \boldsymbol{\sigma}_i$ are the respective dual variables for the consensus power flow constraints (13) to (15). The regularization term are furthermore augumented with penalty factor ρ_i to ensure the asymptotic convergence of ADMM. We present the ADMM steps to solve (16) as follows

1) Local minimization step:

$$\mathbf{p}_i^{\text{exc,*}}, \mathbf{q}_i^{\text{exc,*}}, \mathbf{v}_i^{\text{exc,*}}(k+1) := \arg\min L_i^{\text{admm}}(k) \quad \text{s.t. (16b) to (16e)}$$

The step solves the local version of minimal operation problem while the consensus constraints (13) to (15) being taken into account in the augmented Lagrangian.

2) Global variable update step:

$$\mathbf{p}_i^{\text{exc,+}}(k+1) = \frac{1}{2}(\mathbf{p}_i^{\text{exc,*}}(k+1) - \sum_{j\in\mathcal{N}_i}\mathbf{M}_{ij}\mathbf{p}_j^{\text{exc,*}}) \tag{18}$$

$$\mathbf{q}_i^{\text{exc,+}}(k+1) = \frac{1}{2}(\mathbf{q}_i^{\text{exc,*}}(k+1) - \sum_{j\in\mathcal{N}_i}\mathbf{M}_{ij}\mathbf{q}_j^{\text{exc,*}}) \tag{19}$$

$$\mathbf{v}_i^{\text{exc,+}}(k+1) = \frac{1}{2}(\mathbf{v}_i^{\text{exc,*}}(k+1) - \sum_{j\in\mathcal{N}_i}\mathbf{M}_{ij}\mathbf{v}_j^{\text{exc,*}}) \tag{20}$$

This step requires the information exchange between connected microgrids, i.e., passing $\mathbf{p}_i^{\text{exc,*}}, \mathbf{q}_i^{\text{exc,*}}, \mathbf{v}_i^{\text{exc,*}}$ to neighbors and obtain $\mathbf{p}_j^{\text{exc,*}}, \mathbf{q}_j^{\text{exc,*}}, \mathbf{v}_j^{\text{exc,*}}$ from neighbor $j \in \mathcal{N}_i$.

3) Lagrangian multiplier update step:

$$\boldsymbol{\lambda}_i(k+1) = \boldsymbol{\lambda}_i(k) + \rho_i(\mathbf{p}_i^{\text{exc,*}}(k+1) - \mathbf{p}_i^{\text{exc,+}}(k+1))$$

$$\boldsymbol{\kappa}_i(k+1) = \boldsymbol{\kappa}_i(k) + \rho_i(\mathbf{q}_i^{\text{exc,*}}(k+1) - \mathbf{q}_i^{\text{exc,+}}(k+1)) \tag{21}$$

$$\boldsymbol{\sigma}_i(k+1) = \boldsymbol{\sigma}_i(k) + \rho_i(\mathbf{v}_i^{\text{exc,*}}(k+1) - \mathbf{v}_i^{\text{exc,+}}(k+1)) \tag{22}$$

The ADMM methodology comprises the local minimization step, global variable update and Lagrangian multiplier update. As shown in step 2, the global varibale update requires communication between interconnected neighboring microgrid. In the numerical experiment, we observe the convergence of ADMM is very sensitive to the penalty factor. Hence we adopt the dual balance techniques from [13] to improve the convergence performance. First, the squared primal residual $r_i^2 \in \mathbb{R}$ and dual residual $s_i^2 \in \mathbb{R}$ for microgrid i are defined as follows.

$$r_i^2(k+1) = \|\mathbf{p}_i^{\text{flow},*}(k+1) - \mathbf{p}_i^{\text{flow},+}(k+1)\|_2^2 + \|\mathbf{v}_i^{\text{exc},*}(k+1) - \mathbf{v}_i^{\text{exc},+}(k+1)\|_2^2 + \|\mathbf{q}_i^{\text{exc},*}(k+1) - \mathbf{q}_i^{\text{exc},+}(k+1)\|_2^2 \quad (23)$$

$$s_i^2(k+1) = \|\mathbf{p}_i^{\text{exc},+}(k+1) - \mathbf{p}_i^{\text{flow},+}(k)\|_2^2 + \|\mathbf{v}_i^{\text{exc},+}(k+1) - \mathbf{q}_i^{\text{exc},+}(k)\|_2^2 + \|\mathbf{q}_i^{\text{exc},+}(k+1) - \mathbf{v}_i^{\text{exc},+}(k)\|_2^2 \quad (24)$$

Based on the primal/dual residual results in each ADMM update, the penalty is updated as:

$$\rho_i(k+1) = \begin{cases} \rho_i(k)\cdot(1+\tau) & r_i(k+1) > \eta\, s_i(k+1) \\ \rho_i(k)\cdot(1+\tau)^{-1} & r_i(k+1) < \eta\, s_i(k+1) \\ \rho_i(k) & \text{otherwise} \end{cases}$$

Parameters are commonly set as $\tau = 1, \eta = 10$ [13]. Since problem (16) is a convex optimization problem with a quadratic objective function and linear constraints, ADMM algorithm converges to the global optimum [13].

Remark III.1 (Interpretation of ADMM as a market clearing mechanism)**.** *We demonstrate the equivalence of ADMM as a process of adjusting the prices to achieve the market equilibrium as follows. Each MGO has the initial bids for the commodities of energy and voltage as* $\boldsymbol{\lambda}_i(0), \boldsymbol{\kappa}_i(0), \boldsymbol{\sigma}_i(0)$. *Based on the price information from neighbors, each MGO optimizes the local import/export demand as in the local optimization step of ADMM. Having the demand results obtained in each microgrid, each MGO submits their new demand to their neighbors. Once the new demand is received, each MGO increase the price or reduce the price based on the difference of its supply capability and demand with the step size of* ρ_i, *creating a new bid* $\boldsymbol{\lambda}_i, \boldsymbol{\kappa}_i, \boldsymbol{\sigma}_i$. *The process repeats until a market equilibrium is reached. To this end, the* $\boldsymbol{\lambda}_i, \boldsymbol{\kappa}_i, \boldsymbol{\sigma}_i$ *are the market clear price for exchanged active power, reactive power and voltage support. Therefore, ADMM can be viewed as a fully decentralized market mechanism with neighbor-to-neighbor communications only.*

Remark III.2. *Upon the convergence of ADMM, the prices for active/reactive power and voltage are settled as follows. Consider the illustrative example in Fig. 2 with one coupled bus connecting one seller microgrid and two purchasing microgrids. As for the active/reactive power, the power flows from the seller microgrid to two buyer microgrids with the prices settled as* $\lambda_1 = \lambda_2 = \lambda_3$ *and power injections as:* $p_0 = p_1 + p_2$. *On the other hand, upon market equilibrium, voltage prices follow* $\sigma_0 = \sigma_1 + \sigma_2$, *whereas the voltage magnitude for all microgrids as:* $v_0 = v_1 = v_2$. *This is because the power flows from the upstream microgrid (seller) to the downstream microgrids (buyers). This causes the contribution from the seller microgrid, as compared to buyer microgrids, higher in improving the overall voltage profile of the grid. Hence two buyers procure the voltage with the combined payment from the seller grid based on contributions of coupled bus voltage in the local objective as illustrated in Fig. 2.*

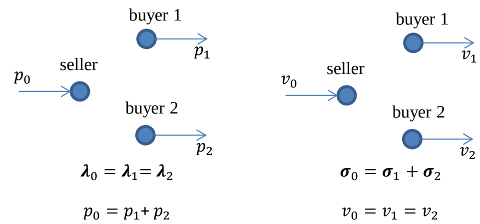

Fig. 2. price settling principle for power and voltage

Table I
TESTNETWORK PARTITION WITH GENERATION PRICE

Microgrid	Included buses	Generation buses	Common buses
1	{1-7;33-87;111}	1;35;52	7
2	{7;88-110}	95	7
3	{7-15;112-134}	130	7;15
4	{15-24;135-138}	138	15;24
5	{24-32;139-141}	32	24

Price vector	DG0(PSP)	DG1	DG2	DG3	DG4	DG5	DG6
P \$/MWh	17	10	13	11	7	10	11
Q \$/Mvarh	3	2	1	2	1.3	2.1	1

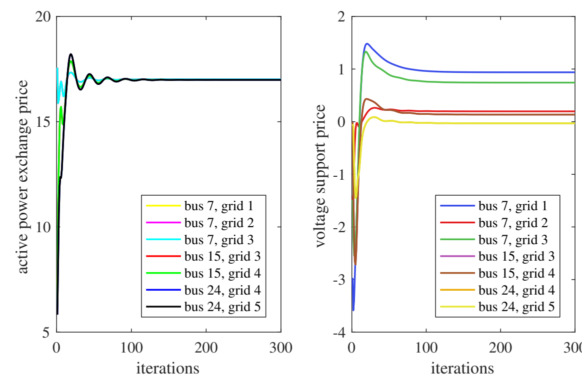

Fig. 3. Case 1: active power price and voltage regulation price ($\mathbf{w} = \mathbf{5}$)

IV. SIMULATION RESULTS

We test the proposed trading framework on a 144-bus distribution network [14] which is partitioned into 5 microgrids. The network partitioning is illustrated in Table I with its associated generation price vectors. Three coupled buses connects the 5 microgrids are bus 7,15 and bus 24. The test network has a total fixed load of 11.9 MW and 7.38 Mvar. In order to demonstrate the efficiency of the proposed method, we consider two realistic scenarios: over-demand and oversupply scenarios. Voltage constraints are kept as $[0.95, 1.05]$; and the ADMM and trust-region parameters are set as: $\eta = 0.1$, $\tau = 0.1$, $\kappa = 10$, $\rho_i(0) = 1.0 \times 10^3, \lambda_i(0) = 0, i \in \mathcal{S}$. The simulations are performed on a personal computer with Intel i5 2.4Ghz and 8 GB RAM.

A. case 1 - over demand scenario

We first consider an over-demand a scenario defined as follows: each DG has a limited generation/injection capacity throughout all the grids, i.e., the collective energy from DGs cannot satisfy the overall demand from all buses. Therefore, the energy consumption in all regions is still partially supplied from the PSP in microgrid 1. From Table I, one can observe that the distributed generations are cheaper than the power obtaining from PSP in microgrid 1. Hence, intuitively, all the

Table II

PRICE RESULTS FOR CASE STUDIES

		bus 7– grid 1	bus 7– grid 2	bus 7– grid 3	bus 15– grid 3	bus 15– grid 4	bus 24– grid 4	bus 24– grid 5
Case 1	Active power price \$/MW	-17	17	-17	-17	17	-17	17
	Reactive power price \$/Mvar	3	-3	-3	3	-3	3	-3
	Voltage price \$/p.u	-0.94	0.20	0.74	-0.13	0.13	0.03	-0.03
Case 2	Active power price \$/MW	17	-17	17	-11	11	-11	11
	Reactive power price \$/Mvar	-3	-3	3	-1	1	-1	1
	Voltage price \$/p.u	-0.23	0	0.23	-0.10	0.1	0	0

-Note the negative sign in the price for import energy/voltage; positive sign for export;

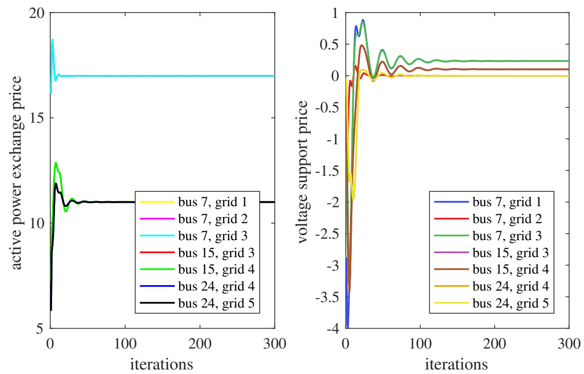

Fig. 4. Case 1: active power price and voltage regulation price ($\mathbf{w} = \mathbf{0}$)

DGs will be fully dispatched to reduce the overall cost. The active power price, therefore, converges to the price at PSP of 17 \$/Mvar which can be verified in Fig. 3. Same explanation applies to reactive power exchange in all microgrids. Now consider the price settlement for voltages illustrated on the right side of Fig. 3 with the price results presented in Table II, the prices of voltages at bus 7 in microgrid 1 equals to the combined the price of bus 7 of microgrid 2 and microgrid 3. Therefore, remark III.2 is verified. The weighting factor for the voltage regulation is set as $\mathbf{w} = \mathbf{5}$ which will be modified in the next test case to show the impacts.

B. case 2 - oversupply scenario

In this test scenario, all the DG generation capacity is increased, i.e., each microgrid can globally seek for the most cost-effective generation sources. In addition, we limit the power import/export in microgrid 3 for both coupled buses at 7 and bus 24 to restrict both active power and reactive power flow. This creates the effect that the power flow from microgrid 1/2 to microgrid 4/5 are "blocked". The microgrids group has practically divided into 2 subgroups, subgroup 1 consisting of microgrids 1/2 and subgroup 2 consisting of microgrid 3/4/5. It leads to the phenomena that the import/export prices converge to two different values as we can observe in Fig. 4. Smart grid 1 and 2 can mainly trade power based on the source of PSP with the price of 17 \$/MWh while microgrid 3/4/5 are trading the power with 11 \$/MWh from the source of DG6. Moreover, the weighting factor of voltage regulation is reduced by $\mathbf{w} = \mathbf{0}$. As a consequence, the coupled bus voltage price has dropped significantly whereas the price settlement remark III.2 remains effective.

V. CONCLUSION AND OUTLOOK

We proposed a novel decentralized market framework for coupled microgrids. Each local microgrid operation regulated the energy flow as well as the voltage trade with the coupled microgrids, preserving its operation autonomy as well as aiding in achieving the overall equilibrium of the system. The numerical validation with the help of multiple scenarios is also presented for the proposed framework.

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