Day-ahead Energy Trade Scheduling for Multiple Microgrids with Network Constraints

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***Abstract*—In this paper, a day-ahead market framework based on alternating direction method of multipliers (ADMM) for trading energy between interconnected microgrids (MGs) is proposed. This framework takes into consideration the electric network constraints along with the inter-temporal battery energy storage systems (BESSs) and instantaneous distributed generators (DGs) power requirements. The market framework allows the interaction between the main grid and the MGs while preserving the local operation independence and autonomy. The framework is tested on a 144-bus network consisting of 3 MGs, and the numerical results validates the effectiveness of the proposed market clearing model.**

***Index Terms*—ADMM, Multiple-MG Operation, BESS, Day-Ahead Market**

I. INTRODUCTION

The advanced MGs with high integration of distributed energy resources (DERs) such as DGs, BESSs and flexible loads are considered as the building blocks of the future smart grid. These MGs can either be connected to the existing distribution grid or be interconnected to form a multiple-MG system. Nevertheless, both technical and economic efficiency must be maintained in the case of energy exchange between MGs as well as the interaction with the main grid. Technically phrasing, this can be considered as maximizing the overall social welfare of the underlying system can be improved while improving the local operation resilience. Hence, market frameworks are needed to address the multiple-MG energy trade problem, i.e., finding the market equilibrium and the correct price signals for MGs to procure and consume energy in an appropriate way.

The smart grid research community has begun to show interest in the market frameworks for MGs [1], [2]. In [3], a distributed energy storage management system with economic dispatch in the multiple-MG scenario is proposed. Assuming the distribution grid to be restructured into a number of smart grid communities, the works [4], [5] propose the energy import and export scheme between the community level as well as between the prosumer level within each community. The state-of-the-art energy trade problem formulation in [2], [3] assume that there are dedicated highways for energy exchange or that MGs are geographically close enough to neglect voltage drop or congestion in the electric grid [4], [5]. The above mentioned works do not consider adequate grid models. This is important as DGs and BESSs are placed closer to consumers, which is usually a low/medium voltage (distribution) grid containing higher losses and nonlinearities [6]. Hence, misrepresentation of network constraints in the problem formulation may lead to either incorrect economic signals or infeasible power flow recovery from the dispatch solution [7]. In [6], [8], using nonlinear programing techniques, the day-ahead market for distribution grid is cleared with adequately represented AC power flow model. However, both works have not achieved the market clearing for MGs in a decentralized way, i.e, market clearing without the presence of a central coordinator.

This work presents a market framework which considers the constraints of both the DERs and the electric grid. The focus is on finding the correct market incentive signals, i.e., the market clearing price for the exchange of active and reactive power between multiple MGs without affecting the electric grid constraints. This is done by incorporating tractable power flow equations into the market clearing mechanism. The framework considers the inter-temporal constraints of BESS and hence is suited for multi-period operation, i.e., the day-ahead market. The ADMM methodology is utilized to achieve the market equilibrium in a decentralized manner. This allows MGs to preserve their private informations such as cost and energy requirements, promoting independence and autonomy of the overall system. Moreover, we show that ADMM can be interpreted as a price adjustment process with the dual variables of the ADMM problem obtained as the market equilibrium price for the respective exchangeable commodities.

Notations: $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex numbers. Non-bold lowercase letters x denote scalars. Bold letters $\mathbf{x}, \mathbf{X}$ denote vectors and matrices. Entries of a matrix $\mathbf{X}$ are specified by x_{ij}. Entries of vector $\mathbf{x}$ are specified by x_i whereas different instances of $\mathbf{x}$ are given as $\mathbf{x}_i$. The obtained optimal solutions are denoted as $x^*, \mathbf{x}^*$. The transpose of a vector or matrix is denoted by $()^\intercal$. Lastly, diag$(\mathbf{x})$ constructs a diagonal matrix with entries of $\mathbf{x}$.

II. PROBLEM SETUP

Consider a number of interconnected MGs operated in connected mode to the main grid (Fig. 1), the MGs comprise distributed generation, energy storage and import/export capabilities with physical connections to other MGs. Each MG is operated by a local microgrid operator (MGO) who plays a supervisory role in monitoring inter-MG transactions. To this end, the local MGO is responsible for clearing inter-MG trade for the day-ahead market and maximizing the social welfare for the given market planning horizon within

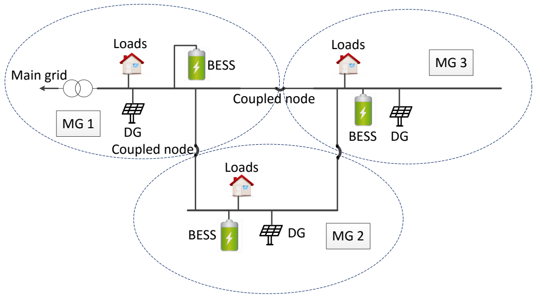

Fig. 1. Multiple MGs with distributed generations and energy storage.

the network constraints. All market participants are assumed to act economically rational and submit their true marginal generation curve to the MGO.

1) Electric grid model: The overall grid consisting of all MGs is assumed to have a distribution grid nature with a reference bus indexed by 0, which is modelled as a slack bus. The rest of the buses in the network are modelled as PQ buses. The set $\mathcal{N} := \{0, 1, 2, ..., n\}$ is defined as the set of all grid buses. The complex nodal injections and voltages are described for all buses with the vectors $\mathbf{s} := \mathbf{p} + \mathrm{j}\mathbf{q}$ and $\mathbf{u} := \mathbf{v} \cdot e^{\mathrm{j}\boldsymbol{\theta}}$ all with size $\mathbb{C}^{n+1}$. We use the positive-semidefinite resistance and reactance matrices $\mathbf{R}, \mathbf{X} \in \mathbb{R}^{(n+1)\times(n+1)}$ to model the grid. Hence, the network power flow is described using the linearized DistFlow model [9]:

$$\mathbf{v} = \mathbf{v_0} + \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} \tag{1}$$

where $\mathbf{v_0} \in \mathbb{R}^{n+1}$ is a vector with all the entries equal to the nominal voltage. Note that the power flow variables are reduced to three variables (i.e., voltage magnitude, active/reactive power injections), since angle variations are assumed to be small across the network for the linearized DistFlow model. In fact, the linearized DistFlow preserves high accuracy of voltage estimation while neglecting the high-order losses in the nonlinear power flow model (at most 0.25% error in the presence of 5% voltage deviation [10]).

2) BESS model: The set $\mathcal{B} := \{1, 2, ..., b\}$ contains the BESSs which are located in all MGs. Let $\mathbf{soc}_t \in \mathbb{R}^b$ denote the state of charge (SOC) of the BESSs at time step t. $\mathbf{Q} \in \mathbb{R}^b$ denotes the battery capacity and $\mathbf{p}^{\mathrm{d}}, \mathbf{p}^{\mathrm{ch}} \in \mathbb{R}^b$ denote the discharge and charge power. The SOC is described with a linearized battery model:

$$\mathbf{soc}_t = \mathbf{soc}_0 - \mathrm{diag}^{-1}(\mathbf{Q}) \sum_{k=1}^{t} (\frac{1}{\eta^d}\mathbf{p}^{\mathrm{d}}_k - \eta^{ch}\mathbf{p}^{\mathrm{ch}}_k)\Delta t \tag{2}$$

where $\eta^{ch}, \eta^d \in \mathbb{R}$ represent the charge and discharge efficiency and Δt represents the market period duration. Hence, the nodal injections of BESS at time t from is given as $\mathbf{p}^{\mathrm{bat}}_t = \mathbf{p}^{\mathrm{d}}_t - \mathbf{p}^{\mathrm{ch}}_t \in \mathbb{R}^b$ as well as $\mathbf{q}^{\mathrm{bat}}_t = \mathbf{q}^{\mathrm{d}}_t - \mathbf{q}^{\mathrm{ch}}_t \in \mathbb{R}^b$.

3) Multiple-MG operation: Consider the topology illustrated in Fig. 1, we use the set $\mathcal{S} := \{1, 2, ..., s\}$ to describe the group of interconnected MGs. The set of buses are partitioned into the respective subsets of $\mathcal{N}$. For example, the local set for the given MG $i \in \mathcal{S}$ is obtained as $\mathcal{N}_i := \{0, 1, 2, ..., n_i\}$ whereas all the generation buses are included in the set $\mathcal{G}_i := \{0, 1, 2, ..., g_i\}$ and BESS in the set $\mathcal{B}_i := \{1, 2, ..., b_i\}$. The time steps for the market planning horizon are included in the set $\mathcal{H} := \{1, 2, ..., h\}$. For the sake of brevity, if a MG is connected to the transmission grid via a power supply point (PSP), this is simply modeled as a generator with the respective locational marginal price, which is cleared by the transmission system operator (TSO). Let $\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t} \in \mathbb{R}^{g_i}$ denote the distributed generations including the power withdrawn from the transmission grid in MG $i \in \mathcal{S}$ at time $t \in \mathcal{H}$, we obtain the total cost of energy generation in MG i at time t:

$$c_{i,t}(\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t}) = \mathbf{c}^{\intercal}_{i,t}\mathbf{p}^{\mathrm{g}}_{i,t} + \mathbf{d}^{\intercal}_{i,t}\mathbf{q}^{\mathrm{g}}_{i,t} \tag{3}$$

where $\mathbf{c}_{i,t}/\mathbf{d}_{i,t} \in \mathbb{R}^{g_i}$ is the generation price vectors for active/reactive power procurement in each market period (see [11] for recent expositions).

For the battery charging and discharging process, an operational cost model considering the battery aging is adapted from [3] resulting in the quadratic cost function:

$$w_{i,t}(\mathbf{p}^{\mathrm{d}}_{i,t}, \mathbf{p}^{\mathrm{ch}}_{i,t}) = (\mathbf{p}^{\mathrm{d}}_{i,t} + \mathbf{p}^{\mathrm{ch}}_{i,t})^{\intercal}\mathrm{diag}(\mathbf{w}_{i,t})(\mathbf{p}^{\mathrm{d}}_{i,t} + \mathbf{p}^{\mathrm{ch}}_{i,t}) \tag{4}$$

with $\mathbf{w}_{i,t} \in \mathbb{R}^{b_i}$ giving the price vector for charge/discharge in each MG.

The intersection between MGs is located at the coupled buses where the MGs can exchange power (see e.g., the coupled buses connecting two neighboring MGs in Fig. 1). We describe the coupled buses for the MG $i \in \mathcal{S}$ with the set $\mathcal{M}_i := \{1, 2, ...m_i\} \in \mathbb{R}^{m_i}$. Furthermore, the power exchange at these coupled buses are denoted as $\mathbf{p}^{\mathrm{exc}}_i, \mathbf{q}^{\mathrm{exc}}_i \in \mathbb{R}^{m_i}$ with its voltage denoted as $\mathbf{v}^{\mathrm{exc}}_i \in \mathbb{R}^{m_i}$. For a MG $i \in \mathcal{S}$, all its physically connected neighboring MGs are included in set $\mathcal{E}_i \subseteq \mathcal{S}$. In particular, the power exchange with MG $j \in \mathcal{E}_i$ and the local MG i is defined as $\mathbf{M}_{ij}\mathbf{p}^{\mathrm{exc}}_j$. The incidence matrix $\mathbf{M}_{ij} \in \mathbb{R}^{m_i \times m_j}$ projects the same coupled bus of MG j to i. Hence, the active/reactive power exchange between the MGs and the coupled voltages at each time instant are constrained by:

$$\mathbf{p}^{\mathrm{exc}}_{i,t} = -\sum_{j\in\mathcal{E}_i} \mathbf{M}_{ij}\mathbf{p}^{\mathrm{exc}}_{j,t} \qquad t \in \mathcal{H} \tag{5}$$

$$\mathbf{q}^{\mathrm{exc}}_{i,t} = -\sum_{j\in\mathcal{E}_{i,t}} \mathbf{M}_{ij}\mathbf{q}^{\mathrm{exc}}_{j,t} \qquad t \in \mathcal{H} \tag{6}$$

$$\mathbf{v}^{\mathrm{exc}}_{i,t} = \mathbf{M}_{ij}\mathbf{v}^{\mathrm{exc}}_{j,t} \qquad \forall j \in \mathcal{E}_i, t \in \mathcal{H} \tag{7}$$

We obtain the nodal injections $\mathbf{p}_i, \mathbf{q}_i \in \mathbb{R}^{n_i+1}$ in MG $i \in \mathcal{S}$ at time $t \in \mathcal{H}$ as the collections of its generations $\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t}$, battery injections $\mathbf{p}^{\mathrm{bat}}_{i,t}, \mathbf{q}^{\mathrm{bat}}_{i,t}$, power exchange (import minus export) and loads $\mathbf{p}^{\mathrm{load}}_{i,t}, \mathbf{q}^{\mathrm{load}}_{i,t} \in \mathbb{R}^{n_i+1}$:

$$\mathbf{p}_{i,t} = \mathbf{A}_i\mathbf{p}^{\mathrm{g}}_{i,t} + \mathbf{B}_i\mathbf{p}^{\mathrm{exc}}_{i,t} + \mathbf{C}_i\mathbf{p}^{\mathrm{bat}}_{i,t} - \mathbf{p}^{\mathrm{load}}_{i,t} \tag{8}$$

$$\mathbf{q}_{i,t} = \mathbf{A}_i\mathbf{q}^{\mathrm{g}}_{i,t} + \mathbf{B}_i\mathbf{q}^{\mathrm{exc}}_{i,t} + \mathbf{C}_i\mathbf{q}^{\mathrm{bat}}_{i,t} - \mathbf{q}^{\mathrm{load}}_{i,t} \tag{9}$$

with $\mathbf{A}_i \in \mathbb{R}^{n_i \times m_i}$ and $\mathbf{B}_i \in \mathbb{R}^{n_i \times g_i}$ represent the mapping matrix from the generation buses and the coupled buses to the grid buses respectively , whereas $\mathbf{C}_i \in \mathbb{R}^{n_i \times b_i}$ is the mapping matrix from the battery connecting buses to the local grid buses.

4) Optimal dispatch problem: To this end, for each local MG $i \in \mathcal{S}$ during the market planning horizon, following operation cost minimization problem applies

$$\min \quad f_i(\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t}) := \sum_{t=1}^{h} \left(c_{i,t}(\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t}) + w_{i,t}(\mathbf{p}^{\mathrm{d}}_{i,t}, \mathbf{p}^{\mathrm{ch}}_{i,t}) \right) \tag{10a}$$

$$\text{s.t.} \quad \mathbf{v}_{i,t} = \mathbf{v}_{0i,t} + \mathbf{R}_i \mathbf{p}_{i,t} + \mathbf{X}_i \mathbf{q}_{i,t} \tag{10b}$$

$$\underline{\mathbf{soc}}_{i,t} = \mathbf{soc}_{i,0} - \mathrm{diag}^{-1}(\mathbf{Q}_i) \sum_{k=1}^{t} \left(\frac{1}{\eta^d}\mathbf{p}^{\mathrm{d}}_{i,k} - \eta^{ch}\mathbf{p}^{\mathrm{ch}}_{i,k}\right)\Delta t \tag{10c}$$

$$\underline{\mathbf{p}}^{\mathrm{g}}_{i,t} \leq \mathbf{p}^{\mathrm{g}}_{i,t} \leq \overline{\mathbf{p}}^{\mathrm{g}}_{i,t} \tag{10d}$$

$$\underline{\mathbf{q}}^{\mathrm{g}}_{i,t} \leq \mathbf{q}^{\mathrm{g}}_{i,t} \leq \overline{\mathbf{q}}^{\mathrm{g}}_{i,t} \tag{10e}$$

$$\underline{\mathbf{p}}^{\mathrm{bat}}_{i,t} \leq \mathbf{p}^{\mathrm{d}}_{i,t} - \mathbf{p}^{\mathrm{ch}}_{i,t} \leq \overline{\mathbf{p}}^{\mathrm{bat}}_{i,t} \tag{10f}$$

$$\underline{\mathbf{soc}}_{i,t} \leq \mathbf{soc}_{i,t} \leq \overline{\mathbf{soc}}_{i,t} \tag{10g}$$

$$\underline{\mathbf{v}}_{i,t} \leq \mathbf{v}_{i,t} \leq \overline{\mathbf{v}}_{i,t} \tag{10h}$$

$$\text{coupled constraints: } (5) - (7)$$

where the operation cost in (10a) is written as the combination of generation cost and the battery operation cost. Constraint (10b) is the regional linearized DistFlow model (1) whereas (10c) is the linearized battery model. Constraints (10d) to (10h) are the box constraints for power dispatch, battery operation and voltage magnitude. Furthermore, (5) and (6) ensure the power supply from neighboring MGs meets the local import demand. Constraint (7) ensures the coupled voltages being the same for all the physically coupled buses. Note that i) the benefit that the MGO receives for supplying the loads is not included in the objective as it is assumed to be fixed and therefore neglected for the sake of simplicity; ii) the market for the energy exchange between MGs is cleared once the problem (10) is solved, i.e., the marginal cost of the energy exchange is obtained.

III. Day-ahead market clearing with ADMM

To preserve the operational autonomy of local MGOs as well as to limit the computational and communication requirements, a decentralized market mechanism which coordinates the solution of (10) and clears the market for energy exchange $\mathbf{p}^{\mathrm{exc}}_i, \mathbf{q}^{\mathrm{exc}}_i$ between the connected MGs is proposed in this work. The goal is twofold: i) to find the market clearing price for the commodities and ii) to obtain the optimal power dispatch scheduling for DGs, BESSs and multi-MG energy exchange. We adopt the ADMM as the solution methodology and provide the economic interpretation of ADMM as a decentralized market clearing process.

In principle, ADMM relies on the augumented Langrangian to bind the coupled constraints and improve the convergence performance. The intuition behind each ADMM iteration is that each MGO solves their local problem (10) and exchanges information with neighbouring MGs to achieve consensus on the coupled constraints (5)-(7). To simplify the notation, we define the power flow state variable vector for MG i at time t as $\mathbf{x}_{i,t} = (\mathbf{p}^{\intercal}_{i,t}, \mathbf{q}^{\intercal}_{i,t}, \mathbf{v}^{\intercal}_{i,t})^{\intercal} \in \mathbb{R}^{3(n_i+1)}$.

Now consider problem (10), if each MGO optimizes the local problem and obtains the results for $\mathbf{x}^{\mathrm{exc}}_{i,t} = ((\mathbf{p}^{\mathrm{exc}}_{i,t})^{\intercal}, (\mathbf{q}^{\mathrm{exc}}_{i,t})^{\intercal}, (\mathbf{v}^{\mathrm{exc}}_{i,t})^{\intercal})^{\intercal} \in \mathbb{R}^{3m_i}$, the connected MG j may not satisfy the desired import demand with its supply capability $\mathbf{M}_{ij}\mathbf{x}^{\mathrm{exc}}_{j,t}$. Therefore, a set of global variables is needed to store the negotiation results for $\mathbf{x}^{\mathrm{exc}}_{i,t}$ after the information exchange with other MGOs. Let $\mathbf{x}^{\mathrm{exc,+}}_{i,t} \in \mathbb{R}^{3m_i}$ denote the global variable, we obtain the augmented Lagrangian for MG $i \in \mathcal{S}$ as:

$$L^{\mathrm{admm}}_i = f_i(\mathbf{p}^{\mathrm{g}}_{i,t}, \mathbf{q}^{\mathrm{g}}_{i,t}) + \sum_{t=1}^{h} \boldsymbol{\lambda}^{\intercal}_{i,t}(\mathbf{x}^{\mathrm{exc}}_{i,t} - \mathbf{x}^{\mathrm{exc,+}}_{i,t}) + \frac{1}{2}\sum_{t=1}^{h} \rho_i (\mathbf{x}^{\mathrm{exc}}_{i,t} - \mathbf{x}^{\mathrm{exc,+}}_{i,t})^{\intercal}(\mathbf{x}^{\mathrm{exc}}_{i,t} - \mathbf{x}^{\mathrm{exc,+}}_{i,t}) \tag{11}$$

where $\boldsymbol{\lambda}_{i,t} \in \mathbb{R}^{3m_i}$ are the respective dual variables for the consensus power flow constraints (5) and (6). The regularization terms are furthermore augumented with penalty factor ρ_i.

1) ADMM iterations: We present the ADMM iterations to solve (10) as follows. At the $(k+1)$-th iteration of ADMM we have the following updates:

$$\mathbf{x}^{\mathrm{exc,*}}_{i,t}(k+1) := \arg\min L^{\mathrm{admm}}_i(k) \quad \text{s.t. (10b) to (10h)} \tag{12}$$

$$\mathbf{x}^{\mathrm{exc,+}}_{i,t}(k+1) = \frac{1}{2}(\mathbf{x}^{\mathrm{exc,*}}_{i,t}(k+1) - \sum_{j\in\mathcal{E}_i} \mathbf{M}_{ij}\mathbf{x}^{\mathrm{exc,*}}_{j,t}(k+1)) \tag{13}$$

$$\boldsymbol{\lambda}_{i,t}(k+1) = \boldsymbol{\lambda}_{i,t}(k) + \rho_i(\mathbf{x}^{\mathrm{exc,*}}_{i,t}(k+1) - \mathbf{x}^{\mathrm{exc,+}}_{i,t}(k+1)) \tag{14}$$

The ADMM-based market clearing comprises the local minimization step, global variable update step and Lagrangian multiplier update step.

The local minimization step solves the local optimal dispatch problem (10) with the consensus constraints (5) to (7) taken into account in the augmented Lagrangian. Then, the global variable update step computes the average value of energy exchange between two connected MGs which requires the information exchange between the neighboring MGs, i.e., passing $\mathbf{x}^{\mathrm{exc,*}}_{i,t}$ to neighbors and obtain $\mathbf{x}^{\mathrm{exc,*}}_{j,t}$ from neighbor $j \in \mathcal{E}_i$ for the planning horizon of h. Note that the global variable update merely requires the neighbor-to-neighbor information exchange with respect to the coupled buses, hence, the operation autonomy is protected since the sensitive information such as load data and biding information are not revealed to other MGOs. Step 3 is the multiplier update step which can be interpreted as a price adjustment step based on the difference between demand $\mathbf{x}^{\mathrm{exc,*}}_{i,t}$ and supply $\mathbf{x}^{\mathrm{exc,+}}_{i,t}$.

2) Convergence properties: The above presented steps adopt the classical consensus-ADMM as the solution methodology. Note that problem (10) is a convex optimization problem that has quadratic objectives with linear constraints, which yields a unique global solution. Hence, the ADMM algorithm converges to the global optimal solution [12]. There exist other variants to extend the classical ADMM to achieve superior convergence. We use the varying penalty (VP) method [12] in this work to measure the convergence error and improve the convergence speed. For the VP method, we first define the squared primal residual $r_i^2 \in \mathbb{R}$ and dual residual $s_i^2 \in \mathbb{R}$ for MG $i \in \mathcal{S}$ as:

$$r_i^2(k+1) = \sum_{t=1}^{h} \|\mathbf{x}_{i,t}^{\text{exc},*}(k+1) - \mathbf{x}_{i,t}^{\text{exc},+}(k+1)\|_2^2 \tag{15}$$

$$s_i^2(k+1) = \sum_{t=1}^{h} \|\mathbf{x}_{i}^{\text{exc},+}(k+1) - \mathbf{x}_{i,t}^{\text{exc},+}(k)\|_2^2 \tag{16}$$

At the $(k+1)$-th iteration, the penalty is updated based on the squared primal/dual residual as:

$$\rho_i(k+1) = \begin{cases} \rho_i(k)\cdot(1+\tau) & r_i(k+1) > \eta\, s_i(k+1) \\ \rho_i(k)\cdot(1+\tau)^{-1} & r_i(k+1) < \eta\, s_i(k+1) \\ \rho_i(k) & \text{otherwise} \end{cases}$$

where $\tau, \eta \in (0,1)$ are the tuning parameters.

Remark III.1 (Interpretation of dual variables of ADMM as a market clearing price)**.** *For the exchange problem defined in* (10)*, the ADMM can be interpreted as a price adjustment process [12], where the dual variables converge to the set of optimal or clearing prices for the exchanged active/reactive power between different MGs. Furthermore, the price settlement follows [12] such that the sum of dual variables for one physical coupled quantity (active power, reactive power, voltage) is equal to 0. Therefore, the following rules of active/reactive power settlements for two connected MGs $i, j \in \mathcal{S}$ and $j \in \mathcal{E}_i$ are obtained as:*

$$\boldsymbol{\lambda}_{i,t} + \mathbf{M}_{ij}\boldsymbol{\lambda}_{j,t} = \mathbf{0} \tag{17}$$

where with respect to the exchanged active/reactive power between two MGs, the market clearing price are the same with opposite signs. A negative price indicates the procurement of energy for the local MG from the neighboring MG increases the local MG operation cost whereas a positive price indicates reward is obtained for the local MG for the amount of energy sold to the neighboring MG.

IV. Results and Discussion

Table I
Test Case Network Parameters

			MG 1		MG 2		MG 3	
Generations	Index	PSP	DG1	DG2	DG3	DG4	DG5	DG 6
	Active power price [\$/MWh]	-	10	13	11	7	10	11
	Reactive power price [\$/Mvarh]	-	2	1	2	1.3	2.1	1
	$\mathbf{p}$ max [MW]	-	1.5	1.2	1.9	1.2	2.3	1.5
	$\mathbf{q}$ max [Mvar]	-	0.3	2.3	0.4	0.6	0.7	0.9
BESSs	Index		BESS 1		BESS 2		BESS 3	
	Battery capacity Q [MWh]		1.9		2.6		1.5	
	Battery aging cost coefficient $\mathbf{w}$		13		10		7	
	Intitial SOC $\mathbf{soc}_{i,0}$		0.3		0.6		0.55	
	Ramping limits $\mathbf{p}^{\text{bat}}$ [MW]		0.06		0.25		0.3	

The proposed market framework is tested on a 144-bus distribution grid [13] which is partitioned into 3 MGs such that $\mathcal{S} := \{1,2,3\}$ and the day-ahead market horizon is defined as $\mathcal{H} := \{1,2,...,24\}$ with 24 market periods. The grid partitioning is illustrated in Fig. 2 where the coupled bus connecting the 3 MGs is located at bus 7. The test network has a total fixed load of 11.92 MW and 7.36 MVAr. The scaled base load time series is generated based on the

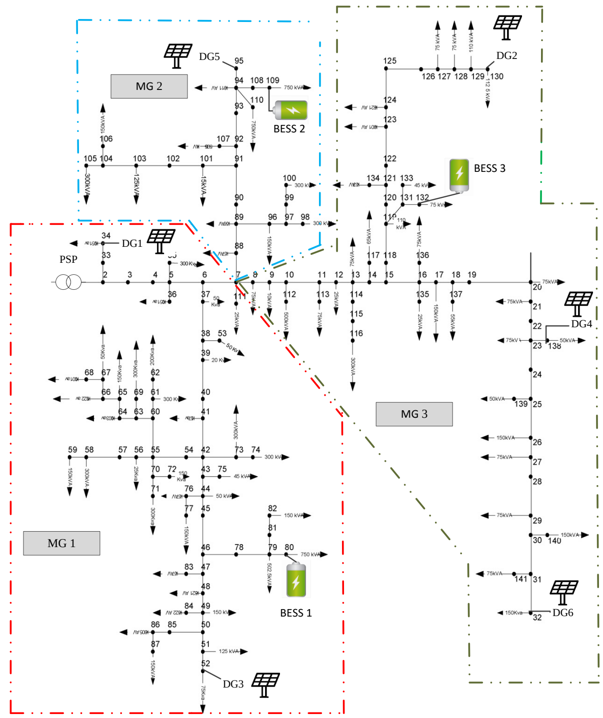

Fig. 2. 144-bus network partitioned into 3 MGs with 6 DGs and 3 BESSs.

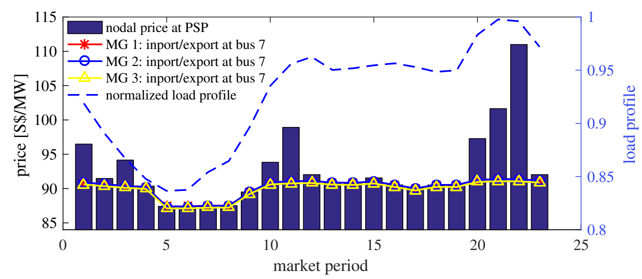

Fig. 3. Nodal price/load profile sample of Singapore's grid are taken from EMCS [14] Singapore on 02/09/2018. The exchange price are recovered from ADMM for the coupled bus for MGs 1-3.

normalized load profile of Singapore [14] (see Fig. 3). In Table I, the parameters for the DGs and BESSs are listed in detail. Voltage constraints are defined as $[0.95, 1.05]$ and the ADMM parameters are set as: $\eta = 0.1$, $\tau = 0.2$, $\kappa = 10$, $\rho_i(0) = 1.0 \times 10^{-3}, \lambda_i(0) = 0, i \in \mathcal{S}$. The charging and discharging efficiency in the battery model are defined as $\eta^{ch} = 0.9, \eta^{d} = 0.9$. The SOC limit is set as $[0.3, 1.0]$. The computations are performed on a computer with Intel i5 2.8 Ghz and 8 GB RAM.

We consider an undersupply scenario defined as follows: Each DG has a limited generation/injection capacity such that the collective supply from all DGs cannot meet the overall demand from all buses (see Table I). Therefore, the PSP in MG 1 still provides energy to meet the energy demand in all MGs. To illustrate the effect of the varying nodal price from the main grid and the impact of energy storage system in improving the total social welfare, we first keep price vector of the DGs constant as in Table I and the nodal price at PSP is taken from the Nation Singapore Electricity Market EMCS

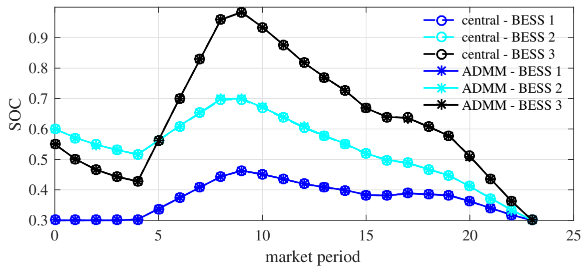

Fig. 4. SOC results: comparison of ADMM to centralized solution.

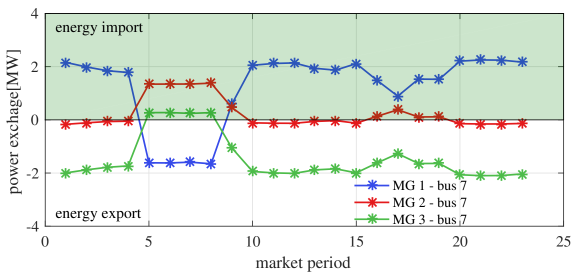

Fig. 5. Import and export power for MG 1-3 on the coupled bus 7 (energy import: positive values; energy export: negative values).

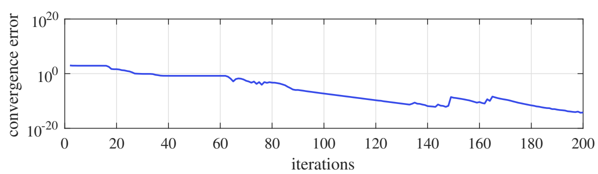

Fig. 6. Convergence error (the sum of primal residual: $\sum_{i=1}^{s} r_i^2$).

(see Fig. 3). Intuitively, the energy exchanged between all three MGs is also supplied by PSP, hence the trading prices are dominated by PSP.

The market clearing price results for the energy exchange between the MGs upon the convergence of ADMM are illustrated in Fig. 3 which demonstrates the dominance of PSP price in the energy trading. It can also be observed that consensus is reached for the energy import/export on the coupled bus. Furthermore, due to the existence of BESS in balancing the overall energy supply and demand, the peak price is shaved when the nodal price at PSP is high. Hence the overall social welfare is improved.

The resulting SOCs of the BESSs are presented in Fig. 4 for ADMM and centralized solution respectively. Since the battery aging cost for BESS 3 is the cheapest, it can be observed that BESS 3 is utilized most intensively to provide energy supply. All three BESSs are discharged when the energy price at PSP and the total demand is high and charged when the energy price from PSP is low. One can also conclude from Fig. 4 that the centralized solution coincides with the distributed solution. The amount of energy which are scheduled to be exchanged during each market period is illustrated in Fig. 5 where the power balance can be observed for the physically coupled bus. The convergence of ADMM is depicted in Fig. 6 with the convergence error, i.e., the sum of primal residuals as in eq. (15) [12].

V. Conclusion

In this work, a multi-period decentralized market framework was proposed for multiple interconnected MGs with consideration of the electric network constraints. The framework regulates the power flow and exchange between connected MGs as well as with the main grid, preserving the operation autonomy of the MGs and improving the overall social welfare. The numerical validation demonstrates the effectiveness of the proposed framework.

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