Scenario Based N-1 Transmission Expansion Planning using DC Mixed Integer Programming

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***Abstract*—This paper presents a scenario based mixed integer programming method to solve the transmission expansion planning problem with security constraints. To consider the N-1 line outages, the scenarios are introduced as a predefined set of parameters in the mathematical model. A DC power flow model is used as a simplified approach to represent the electrical system. The proposed expansion planning model is applied to the well known Garver 6 bus test system and a realistic 46 bus Brazilian test system. The versatility of the proposed method gives the freedom to test the expansion planning as a complete one-stage approach and as the common two-stage approach. The simulation results are presented to draw a conclusion of the performance of the proposed method.**

***Index Terms*—Power System Planning, Power Transmission, Mixed Integer Programming, Optimization, DC Power Flow, Security Constraints.**

NOMENCLATURE

Sets

Ω_l	Set of lines	
Ω_n	Set of nodes	
Ω_g	Set of generator nodes	
Ω_{sc}	Set of number of lines scenarios	
Ω_{fix}	Set of pre-existing lines	
Ω_o	Set of outage lines	
Constants		
X_{ij}	Line reactance	$[\Omega]$
P_i^d	Node active power demand	$[MW]$
P_i^{gmin}	Minimum generation of active power	$[MW]$
P_i^{gmax}	Maximum generation of active power	$[MW]$
P_{ij}^{max}	Maximum line power flow	$[MW]$
Continuous variables		
P_{ij}	Active power flow of branch ij	$[MW]$
P_i^g	Active power generated at node i	$[MW]$
θ_i	Phase angle at node i	
β_{ij}	Auxiliary variable for branch ij	
Binary variables		
α_{ij}	Decision variable for line ij	

I. INTRODUCTION

Recent technological breakthroughs have introduced significantly more complex power systems with a constant increase in energy demand. To ensure a normal power system operation, it is important to use transmission system expansion planning methods to determine an optimal network expansion plan with minimal investment costs.

In the literature, numerous research efforts are made in this domain. Due to the complex nature of the problem, a static long term expansion planning for a single point in time in the future is the most widely researched and accepted approach [1]-[3]. Different methods are reported in the literature, extending to include various applications. A common application is to account for an energy market environment and market clearing problems with consideration of the social welfare [4]. Other works may consider probabilistic methods to account for uncertainties such as load variations, generator rescheduling, market competitions and availability of system facilities [5]. With the increase in renewable energy generation, recent work includes methods that consider various renewable energy resources and energy storage systems [5]-[7].

In most expansion planning research work, the security criteria of the system is not considered. This means that the obtained optimal solution does not account for contingencies caused by the outage of a transmission line, known as the N-1 criteria. The ability of a power system to preserve the normal state during N-1 outage is a universally accepted fundamental security criteria for system operation [8].

Some contribution in expansion planning that considers the N-1 security criteria can be found in [9]-[13]. A common approach for security constraint expansion planning is to use a two-stage optimization process [9], [10]. In the 1st stage, mixed integer programming (MIP) is used to obtain a basic expansion planning of the network. Using the same strategy, the 2nd stage accounts for the line outage and extends the solution to a N-1 satisfactory network. The two stage approach proves to be computationally efficient and reliable. However, the 1st stage is directly influencing the final solution, often resulting in a non optimal plan.

Due to the heuristic nature of the problem, metaheuristic methods are widely accepted as an alternative to multi-stage MIP optimization. The authors in [12] introduce an improved genetic algorithm that gives a better optimal solution when compared to a two-stage optimization. Other heuristic methods may include taboo search, simulated annealing and particle swarm optimization [13]. The main disadvantage of heuristic methods is the relatively large computational effort.

In this paper, a static scenario based mixed integer programming (SC-MIP) method for transmission expansion planning with security constraints is proposed. The scenarios are introduced as a predefined set of parameters, which gives a versatility to extend the model for different applications.

In order to meet the security criteria, the presented mathematical model considers the N-1 line outage. However, the method can also be extended to consider the outage of generators and transformers. A lossless DC power flow model is used as a simplified approach to represent the electrical system. Additionally, the presented method can be used in more accurate models like the AC model. The proposed model can be solved to optimality by using the existing MIP solvers such as CPLEX.

The proposed SC-MIP expansion planning model is applied to the well known Garver 6 bus test system and a realistic 46 bus South Brazilian test system [1], [14]. The versatility of the proposed method gives the freedom to test the expansion planning as a complete one-stage approach and as the common two-stage approach. The results show that the optimal solution is obtained with a great computational efficiency when compared to the meta-heuristic methods. The computational efficiency is better when a two-stage approach is used, but the one-stage approach gives a better optimal solution.

II. Transmission Expansion Planning

The transmission system expansion planning can be defined as a mixed integer problem as follows

$$\min \sum_{(i,j)\in\Omega^l} \Psi_{ij} \cdot \alpha_{ij} \tag{1}$$

subject to

$$\sum_{(j,i)\in\Omega_l} P_{ji} - \sum_{(i,j)\in\Omega_l} P_{ij} + P_i^g = P_i^d \qquad \forall i \in \Omega_n \tag{2}$$

$$P_{ij} = \frac{1}{X_{ij}} \cdot (\theta_j - \theta_i) \qquad \forall (i,j) \in \Omega_l \tag{3}$$

$$0 \le |P_{ij}| \le P_{ij}^{max} \cdot \alpha_{ij} \qquad \forall (i,j) \in \Omega_l \tag{4}$$

$$0 \le |P_i^g| \le P_i^{gmax} \qquad \forall i \in \Omega_g \tag{5}$$

$$-\frac{\pi}{2} \le |\theta_j - \theta_i| \le \frac{\pi}{2} \qquad \forall (i,j) \in \Omega_l \tag{6}$$

$$\theta_{ref} = 0 \tag{7}$$

$$\alpha_{ij} = 1 \qquad \forall (i,j) \in \Omega_{fix} \tag{8}$$

The objective function defined with (1) is a cost function that minimizes the investment cost for transmission line expansion of the grid. Equation (2) is a power balance constraint that ensures a zero sum of the generated, consumed and transmitted power at each node. A DC power flow is defined with equation (3). The power flow limits of a branch ij are then set using constraint (4). The maximum generator capacity at generating nodes is fixed with (5). The grid's operating stability is maintained by limiting the phase angle difference using constraint (6). Lines that exist in the grid before the expansion planning are fixed with (8). In order to bound the phase angle, a reference phase angle at a node is set to zero in (7). It is most common that the biggest generator is chosen as a reference node.

The phase angle θ_i is a continuous variable existing for each node of the set Ω_n. This means that the phase angle difference $(\theta_j - \theta_i)$ will be calculated even when no line in branch ij exist. This state does not allow the power flow of branch ij in (3) to be set to zero, which makes the problem infeasible. To solve the infeasibility, an auxiliary variable $\beta_{i,j}$ is introduced and the equation (3) is replaced with equation (9) as follows

$$P_{ij} = \frac{1}{X_{ij}} \cdot (\theta_j - \theta_i) + \beta_{ij} \qquad \forall (i,j) \in \Omega_l \tag{9}$$

$$0 \le |\beta_{ij}| \le P_{ij}^{max} \cdot (1 - \alpha_{ij}) \qquad \forall (i,j) \in \Omega_l \tag{10}$$

The auxiliary variable is assigned the value of the angle difference between two nodes i and j whenever the line in ij is not selected and $\alpha_{ij} = 0$. When a line exist and $\alpha_{ij} = 1$, the auxiliary variable is not considered and it is set to zero using constraint (10).

However, the above expansion planning model is limited to consider only one line for each branch ij. If multiple parallel lines in a branch ij are considered, the resultant reactance X_{ij} in the power flow calculation changes accordingly as follows

$$X_{ij}^* = \frac{X_{ij}{}^{\tau_{ij}}}{\tau_{ij} \cdot X_{ij}{}^{(\tau_{ij}-1)}} \tag{11}$$

where τ_{ij} is an integer variable that represents the total number of parallel lines in the branch ij. Then, the DC power flow equation (3) becomes

$$P_{ij} = \frac{1}{X_{ij}^*} \cdot (\theta_j - \theta_i) \qquad \forall (i,j) \in \Omega_l \tag{12}$$

The DC power flow definition in (12) becomes very complex and non-linear, which makes it very difficult to solve. To avoid such non-linearity when multiple lines are considered, a scenario based approach is introduced.

The proposed scenario based transmission expansion planning is defined as follows

$$\min \sum_{(i,j)\in\Omega_l, s\in\Omega_{sc}} \Psi_{ij,s} \cdot \alpha_{ij,s} \tag{13}$$

subject to

$$\sum_{(j,i)\in\Omega_l, s\in\Omega_{sc}} P_{ji,s} - \sum_{(i,j)\in\Omega_l, s\in\Omega_{sc}} P_{ij,s} + P_i^g = P_i^d \qquad \forall i \in \Omega_n \tag{14}$$

$$P_{ij,s} = \frac{1}{X_{ij,s}} \cdot (\theta_j - \theta_i) + \beta_{ij,s} \quad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc} \tag{15}$$

$$0 \le |\beta_{ij,s}| \le P_{ij,s}^{max} \cdot (1 - \alpha_{ij,s}) \quad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc} \tag{16}$$

$$0 \le |P_{ij,s}| \le P_{ij,s}^{max} \cdot \alpha_{ij,s} \qquad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc} \tag{17}$$

$$\sum_{s\in\Omega_{sc}} \alpha_{ij,s} \le 1 \qquad \forall (i,j) \in \Omega_l \tag{18}$$

$$\sum_{s\in\Omega_{sc}} \alpha_{ij,s} = 1 \qquad \forall (i,j) \in \Omega_{fix} \tag{19}$$

A new set of scenarios Ω_{sc} is defined. The number of scenarios corresponds to the maximum allowed number of lines per branch. The reactance $X_{ij,s}$ and the cost of the lines $\Psi_{ij,s}$ are pre-calculated for each scenario s accordingly. Similarly, the maximum power carrying capacity $P_{ij,s}^{max}$ is predefined for each scenario s. For lines that already exist in a branch, the cost is set to zero. The cost is considered only for additional and non-existing lines. A binary variable for each branch ij is then assigned to each scenario s. Based on the binary decision variable during optimization, a power flow is calculated for each of the selected scenarios.

An additional constraint (18) is defined. This constraint ensures that each branch ij has a maximum of one scenario selected. Constraints (5)-(7) are included, but not subjected to the newly defined scenarios. The transmission expansion planning defined with (13)-(19), (5)-(7) is a mixed integer programming optimization and can be efficiently solved with most conventional solvers that guarantee the optimality.

III. N-1 Transmission Expansion Planning

The proposed scenario based mathematical model is extended to meet the N-1 security constraint within the expansion planning optimization as follows

$$\min \sum_{(i,j)\in\Omega^l, s\in\Omega_{sc}} \Psi_{ij,s} \cdot \alpha_{ij,s} \tag{20}$$

subject to

$$\sum_{(j,i)\in\Omega_l, s\in\Omega_{sc}} P_{ji,s}^{(l)} - \sum_{(i,j)\in\Omega_l, s\in\Omega_{sc}} P_{ij,s}^{(l)} + P_i^g = P_i^d \qquad \forall i \in \Omega_n, \forall l \in \Omega_o \tag{21}$$

$$P_{ij,s}^{(l)} = \frac{1}{X_{ij,s}^{(l)}} \cdot \left(\theta_j^{(l)} - \theta_i^{(l)}\right) + \beta_{ij,s}^{(l)} \qquad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc}, \forall l \in \Omega_o \tag{22}$$

$$0 \leq \left|\beta_{ij,s}^{(l)}\right| \leq P_{ij,s}^{(l)max} \cdot (1 - \alpha_{ij,s}) \qquad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc}, \forall l \in \Omega_o \tag{23}$$

$$0 \leq \left|P_{ij,s}^{(l)}\right| \leq P_{ij,s}^{(l)max} \cdot \alpha_{ij,s} \qquad \forall (i,j) \in \Omega_l, \forall s \in \Omega_{sc}, \forall l \in \Omega_o \tag{24}$$

$$\sum_{s\in\Omega_{sc}} \alpha_{ij,s} \leq 1 \qquad \forall (i,j) \in \Omega_l \tag{25}$$

$$\sum_{s\in\Omega_{sc}} \alpha_{ij,s} = 1 \qquad \forall (i,j) \in \Omega_{fix} \tag{26}$$

$$0 \leq |P_i^g| \leq P_i^{gmax} \qquad \forall i \in \Omega_n \tag{27}$$

$$-\frac{\pi}{2} \leq \left|\theta_j^{(l)} - \theta_i^{(l)}\right| \leq \frac{\pi}{2} \qquad \forall (i,j) \in \Omega_l, \forall l \in \Omega_o \tag{28}$$

$$\theta_{ref}^{(l)} = 0 \qquad \forall l \in \Omega_o \tag{29}$$

According to the N-1 security criterion, the grid shall continue to normally operate without rescheduling the generation dispatch following the loss of a transmission line. This criterion is met by introducing a new scenario set Ω_o. A different line is switched off in each of the scenarios defined by the set Ω_o. This is done such that the power flow of branch ij for scenario s is constrained to correspond to loss of a single line for each of the scenarios l. The input parameter $P_{ij,s}^{(l)max}$ in (24) is predefined accordingly. The reactance $X_{ij,s}^{(l)}$ in (22) is pre-calculated to meet the newly occurred situation in the branch ij for scenario s in which a line from scenario l is switched off. When only one line in a branch exists, the reactance is set to a very high value that will replicate infiniteness and set the power flow in (22) to zero in addition to the constraint $P_{ij,s}^{(l)max} = 0$ in (24).

As generation dispatch is not allowed, the node generation variable P_i^g is not affected by any of the scenarios. Since the scenarios are set by the line limit parameter and the reactance parameter, the binary variable does not increase in complexity and does not consider the N-1 scenarios. However, the power flow calculation needs to be considered for each of the N-1 scenarios in order to prove a feasible and optimal N-1 solution. The binary variable as a decision variable gives a solution that will be feasible for each of the line outages defined with set Ω_o. The current stage of the proposed model is suitable for small and medium sized grids as shown in the case study results. Due to the increase of variables, solving large systems may require extensive solving time.

IV. Case Study and Results

The proposed definition for transmission system expansion planning with and without security constraints is tested on two different case studies. The first case study is a 6 bus transmission network proposed by Garver and the second case study is a South Brazilian 46 bus transmission netwok. Detailed information including the electrical system data are found in [1] and [14].

When defining the scenarios for the N-1 SC-MIP expansion planning, two different approaches are used. At one approach, the scenarios are defined as a 2nd-stage optimization considering the optimal configuration obtained from a 1st-stage SC-MIP expansion planning as an initial network. The other approach is to run the N-1 SC-MIP expansion planning as a one-stage optimization, independently and without considering any initial solution but the original existing network.

A. 6 Bus Garver System

The existing network with load an generator data of the 6 bus Garver system is show in Fig. 1.

Two different scenarios are considered for testing the optimal expansion planning and the optimal security constraint expansion planning. The scenarios used as case studies are detailed in [12].

1) Existing network without generation rescheduling: The proposed SC-MIP expansion planning method finds an optimal solution for the investment cost of \$200 000 when generation

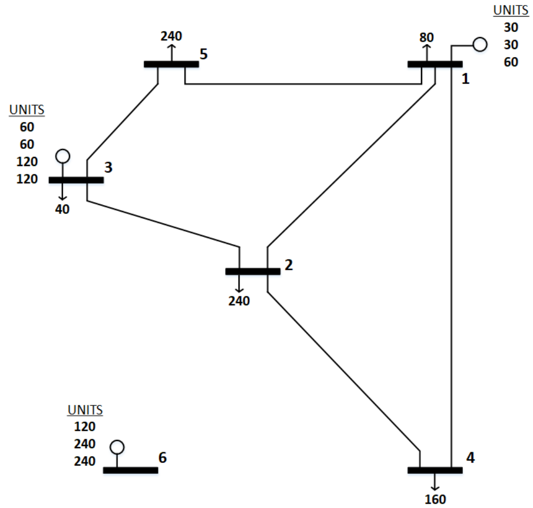

Fig. 1. Garver 6 bus existing network with loads and generating units

rescheduling is not allowed. The existing network is expanded by adding the following lines: n_{2-6}=4, n_{3-5}=1 and n_{4-6}=2.

The N-1 SC-MIP expansion planning finds an optimal solution of $298 000 for the investment cost. The security constraints are met by expanding the existing network with the following lines: n_{2-6}=4, n_{3-5}=2, n_{3-6}=1 and n_{4-6}=3. The same solution was obtained for both N-1 MIP approaches.

2) Existing network with generation rescheduling: The obtained optimal solution for the investment cost is $110 000 when generation rescheduling is allowed. The existing network is expanded by adding the following lines: n_{3-5}=1 and n_{4-6}=3.

The N-1 SC-MIP finds an optimal solution for the investment cost of $180 000. The following lines are added: n_{2-3}=1, n_{2-6}=1, n_{3-5}=2 and n_{4-6}=3. The same solution was obtained for both N-1 SC-MIP approaches.

B. 46 Bus South Brazilian System

The 46 bus South Brazilian system is shown on Fig. 2, with dotted lines representing the potential expansion lines.

The proposed method obtains an optimal solution of $72 million investment cost when no security constraints are considered. The following lines are added: n_{2-5}=1, n_{5-6}=2, n_{13-20}=1, n_{20-21}=2, n_{20-23}=1, n_{42-43}=1 and n_{46-6}=1.

The obtained solution is then considered as an initial network for a 2nd-stage N-1 SC-MIP expansion planning. The total investment cost for the expansion planning with security constraints is $220 million. The 1st-stage optimal solution is expanded by adding the following lines: n_{5-6}=1, n_{12-14}=1, n_{19-21}=1, n_{20-21}=1, n_{20-23}=1, n_{31-32}=1, n_{32-43}=1, n_{33-34}=1, n_{42-43}=2, n_{42-44}=1 and n_{44-45}=1.

When the initial 1st-stage is not considered, the N-1 SC-MIP expansion planning results with an optimal configuration with investment cost of $213 million. The following lines are added: n_{2-5}=1, n_{5-6}=3, n_{12-14}=1, n_{19-21}=1, n_{20-21}=3, n_{20-23}=2, n_{31-32}=1, n_{32-43}=1, n_{42-43}=2, n_{42-44}=1, n_{44-45}=1 and n_{46-06}=2.

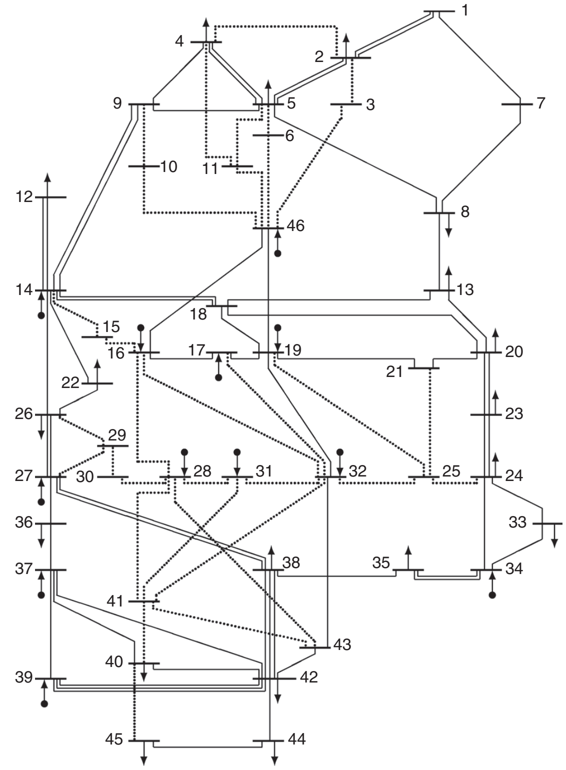

Fig. 2. South Brazilian 46 bus system with 79 branches

V. Results Analysis

Testing the proposed expansion planning model on the 6 bus Garver system shows results that match the optimal solutions reported in the literature [15]. The proposed security constraint extension planning gives the same optimal solution found in [9], [12]. When the security constraints are considered, the results obtained for a two-stage optimization are no different than the one-stage N-1 SC-MIP expansion planning approach.

Similarly, the results obtained for the bigger 46 bus network also match the optimal results reported in the literature for both expansion planning methods, with and without security constraints [12]. However, the results differ when a two-stage N-1 optimization is considered. Due to the fixed lines of the initial stage, the 2nd-stage N-1 solution consists of increased number of lines which results in an investment cost higher than the optimal solution.

TABLE I
Computational effort for case studies in seconds

	Garver No Resch	Garver Resch	46 Bus South Brazilian
SC-MIP	0.20	0.36	7.94
N-1 SC-MIP 2nd-stage	1.46	1.54	7200
N-1 SC-MIP	2.14	2.22	16800

The computational efficiency of the proposed method is summarized in Table I. The proposed method has shown great efficiency for small and medium size grids when compared to metaheuristic methods [12]. The two-stage N-1 expansion planning proves to be a more computational efficient approach when compared to the overall one-stage N-1 expansion planning. However, the result can differ from the optimal solution as shown in the 46 bus case study.

Increasing the number of nodes significantly increases the complexity and the required computational time. To solve this problem, new unconventional methods for solving mixed integer programming problems are researched and developed. Using graphics processing units (GPUs) instead of the central processing units (CPUs) or hybrid solutions are highly recommended when large systems are considered [16].

VI. Conclusion

A mathematical model for scenario based mixed integer programming method is proposed to solve the transmission expansion planning with security constraints. The scenario based approach gives a versatility to extend the model for different applications. Besides considering the N-1 line outage, the model has an advantage that the scenarios can be defined to consider N-1 generator and transformer outage.

A DC power flow model is used to represent the electrical system. However, further research will be focused to implement and test the presented method with more accurate models such as a DC lossy and an AC model. The advantage of the proposed model is that it can be used as a complete one-stage optimization problem besides the common two-stage approach. It is defined as a MIP problem that can be solved to optimality by using existing MIP solvers such as CPLEX.

Results of the performance of the model applied to two well known small and medium test systems are shown. The two-stage approach shows better computational efficiency, but the results can differ from the optimal solution obtained with the one-stage optimization. Even though the proposed method shows better computational efficiency when compared to metaheuristic methods, the complexity increases with the size of the system. Due to the high computational effort for large systems, a further research in using GPU or hybrid computing solutions needs to be conveyed.

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